Expander2

– a formal methods presenter and animator –

September 3, 2007

Expander2 is a flexible multi-purpose workbench for interactive term rewriting, graph transformation, theorem proving, constraint solving, flow graph analysis and other procedures that build up proofs or computation sequences. Moreover, tailor-made interpreters display terms as two-dimensional structures ranging from trees and graphs to a variety of pictorial representations that include tables, matrices, alignments, partitions, fractals and various tree-like or rectangular graph layouts (see *Widget interpreters*). Proofs and computations performed with Expander2 follow the rules and the semantics of *swinging types*. Swinging types are based on many-sorted predicate logic and combine constructor-based types with destructor-based (e.g. state-based) ones. The former come as *initial* term models, the latter as *final* models consisting of context interpretations. Relation symbols are interpreted as least or greatest solutions of their respective axioms.

The user may interact with the system at three levels of decreasing control over proofs and computations. At the top level, rules like induction and coinduction are applied locally and step by step. At the medium level, goals are rewritten or narrowed, i.e. axioms are applied exhaustively and iteratively. At the bottom level, built-in rules (some of them executing Haskell programs) simplify, i.e. (partially) evaluate terms and formulas, and thus hide routine steps of a proof or computation (see *Overview*). Proofs are automatically translated into proof terms that can be evaluated and modified later. This allows one to design functional-logic programs as *proof carrying code* that a client can validate by running the proof term evaluator (*proof checker*).

Expander2 has been written in O'Haskell, an extension of Haskell with object-oriented features for reactive programming and a typed interface to Tcl/Tk. Besides a comfortable GUI the design goals of Expander2 were to integrate testing, proving and visualizing deductive methods, admit several degrees of interaction and keep the system open for extensions or adaptations of individual components to changing demands.

Send comments, bugs, etc. to Peter Padawitz. *Any suggestions for improvements, extensions, applications or project proposals are welcome!*

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Main commands

A command followed by a letter in round brackets is executed when the corresponding key is pushed after the keyboard has been activated by placing the cursor over the entry resp. label field and pressing the left mouse button. The keys for *add spec*, *apply clause*, *load text* and *save tree* work if the entry field has been activated. The keys for *parse up* and *parse down* work if the text field has been activated. The keys for other commands work if the label field has been activated.

add map	add spec from file (Return)	apply for symbols
apply clause (a/b) left to right (Left) - right to left (Right)	apply (axioms) in text field	apply map
apply substitution	apply to variable:	build equations
build graph	build list	build Trans/TransL
build unifier	call enumerator	check proof term (c)
clear subtrees	coinduction	collapse
collapse levelwise	coordinates	copy
create Hoare invariant	create induction hypotheses	create subgoal invariant
decompose atom	decrease current (Left)	derive/stop
enclose/replace by text	expand	fixpoint induction
flatten (co-)Horn clause	generalize	heap numbers
height numbers	hide/show	Horn axioms for copredicates
increase current (Right)	instantiate	invert for symbols
label graph with Atoms	label roots with entry	load text from file (Up)

move up quantifiers (m)	narrow/rewrite (n)	negate for symbols
paint	parse up (Up)	parse down (Down)
polarities	positions	preorder numbers
redraw (z)	remove entry&label	remove from entry field
remove other trees	re-add/remove spec	remove subtrees
remove text	rename	replace by other sides
replace by tree of Solver1/2	reverse (r)	save spec to file
save proof to file	save proof term to file (p)	save tree to file (Down)
save tree in eps format to file (i)	shift subformulas	set
show axioms for symbols (x)	show changed	show map
show node infos	show sig	simplify (s)
split/join	store graph	stretch conclusion
stretch premise	subsume	turn local def into function application
unify	unify with tree of Solver1/2	use transitivity
<---/---> (Down/Up)	+1/-1	

Create two subdirectories of your home directory and call them *Examples* and *Pics*, respectively. The *save* commands of Expander2 store strings into files of *Examples* and graphs (in *eps* format) into files of *Pics*. If the directories do not exist, nothing is saved! A file parameter of *add* or *load* commands is looked up first in your *Examples* directory. If it is not found there, it is searched for in the synonymous system directory. *Gif* files used as widgets (see *Widget interpreters*) are also looked up in the *Examples* directory.

Overview

The main components of Expander2 are the **solver**, the **painter**, the **simplifier**, the **enumerator** and a **recorder** of proofs and computation sequences.

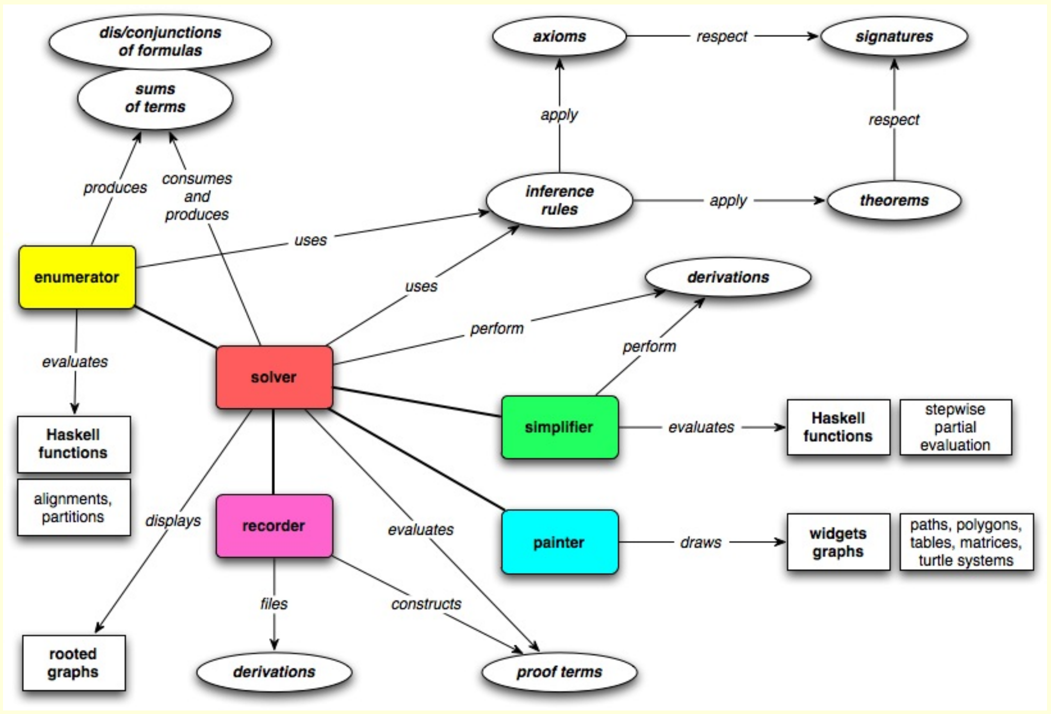


Fig. 1. *Components of Expander2*

The **solver** is accessed via a window for editing and displaying trees that represents a disjunction or conjunction of logical formulas or a sum of functional terms. A proper (non-singleton) sum results from a computation obtained by nondeterministic rewriting. The solver window has a canvas for the two-dimensional representation of the list of current trees (among which one browses by moving the slider below the window) and a text field for their string representation. With the *parse buttons* one switches between the tree (or graph) and the string representation. Both representations are editable. As the usual cut, copy and paste operate on substrings in the text field, so do corresponding mouse-triggered functions when the cursor is moved over subtrees on the canvas.

After a *widget interpreter* has been selected from the *pict type* menu, pushing the *paint* button opens a *painter* window and the pictorial representations of all interpretable subtrees of the solver's current trees will be shown. *Pictures* are lists of *widgets* that can be edited in the painter window and completed to *widget graphs*. Widgets are built up of path, polygon and *turtle action* constructors that admit the definition of a variety of pictorial representations ranging from tables and matrices via string alignments, piles and partitions to complex fractals generated by *turtle systems* [RS], which define a picture in terms of a sequence of actions that a turtle would perform when drawing the picture while moving over a canvas. The turtle works recursively in two ways: it maintains a stack of positions and orientations where it may return to, and it may give birth to subturtles, i.e. call other turtle systems. The solver and its associated painter are fully synchronized: the selection of a tree in the solver window is automatically translated to a selection of the tree's pictorial representation in the painter window and vice versa. Hence rewriting, narrowing and simplification steps can be carried out from either window.

The **enumerator** provides algorithms that enumerate trees or graphs and passes their results both to the solver and the painter.

Currently, two algorithms are available: a generator of all sequence alignments [Gie,P01] satisfying constraints that are partly given by axioms, and a generator of all nested partitions of a list with a given length and satisfying constraints given by particular predicates. The painter displays an alignment in the way DNA sequences are usually visualized. A nested partition is displayed as the corresponding rectangular dissection of a square.

Expander2 allows the user to control proofs and computations at **three levels of interaction**.

At the high level, analytic or synthetic inference rules or other syntactic transformations are applied individually and locally to selected subtrees (see the *transform-selection menu*). The rules cover single axiom applications, substitution or unification steps, Noetherian, Hoare, subgoal or fixpoint induction and coinduction. Derivations are correct if, in the case of trees representing terms, their sum is equivalent to the sum of their sucessors or, in the case of trees representing formulas, their dis- resp. conjunction is implied by the dis- resp. conjunction of their successors. The underlying models are determined by built-in data types and the least/greatest interpretation of Horn/co-Horn axioms. Incorrect deduction steps are detected and cause a warning. All proper tree transformations are recorded, be they correct proofs or other transformations. Terms and formulas are built up from the symbols of the current signature (see *Solver state variables*). For more details on the syntax and semantics of axioms, theorems and goals, see *Axioms and theorems* and Swinging Types.

At the medium level, rewriting and narrowing realize the iterated and exhaustive application of all axioms for the defined functions, predicates and copredicates of the current signature. Terminating rewriting sequences end up with **normal forms**, i.e. terms consisting of constructors and variables. Terminating narrowing sequences end up with the formula `True`, `False` or solved formulas that represent solutions of the initial formula. Since the axioms are functional-logic programs in abstract logical syntax, rewriting and narrowing agree with program execution. Hence the medium level allows one to test such programs, while the inference rules of the high level provide a "tool box" for program verification. In the case of finite data sets, rewriting and narrowing is often sufficient even for program verification. Besides relations and deterministic functions, non-deterministic transition systems employing structured states, such as *Maude programs* [C] or algebraic nets [SMÖ], may also be axiomatized and verified by Expander2. The latter are executed by applying associative-commutative rewriting or narrowing on *bag terms*, i.e. multisets of terms.

At the low level, built-in Haskell functions simplify or (partially) evaluate terms and formulas and thereby hide most routine steps of proofs or computations. The functions comprise arithmetic, list, bag and set operations, term equivalence and inequivalence (that depend on the current signature's constructors) and logical simplifications that turn formulas into *nested Gentzen clauses*. Evaluating a function f at the medium level means narrowing upon the axioms for f, Evaluating f at the low level means running a built-in Haskell implementation of f. This allows one to test and debug algorithms and visualize their results. For instance, translators between different representations of Boolean functions were integrated into Expander2 in this way. In addition, an execution of an iterative algorithm can be split into its loop traversals such that intermediate results become visible, too. Currently, the computation steps of Gaussian equation solving, automata minimization [HMU], OBDD optimization, LR parsing, data flow analysis and global model checking can be carried out and displayed (see *Simplifications*).

Overall code structure

The code of Expander2 consists of four O'Haskell modules:

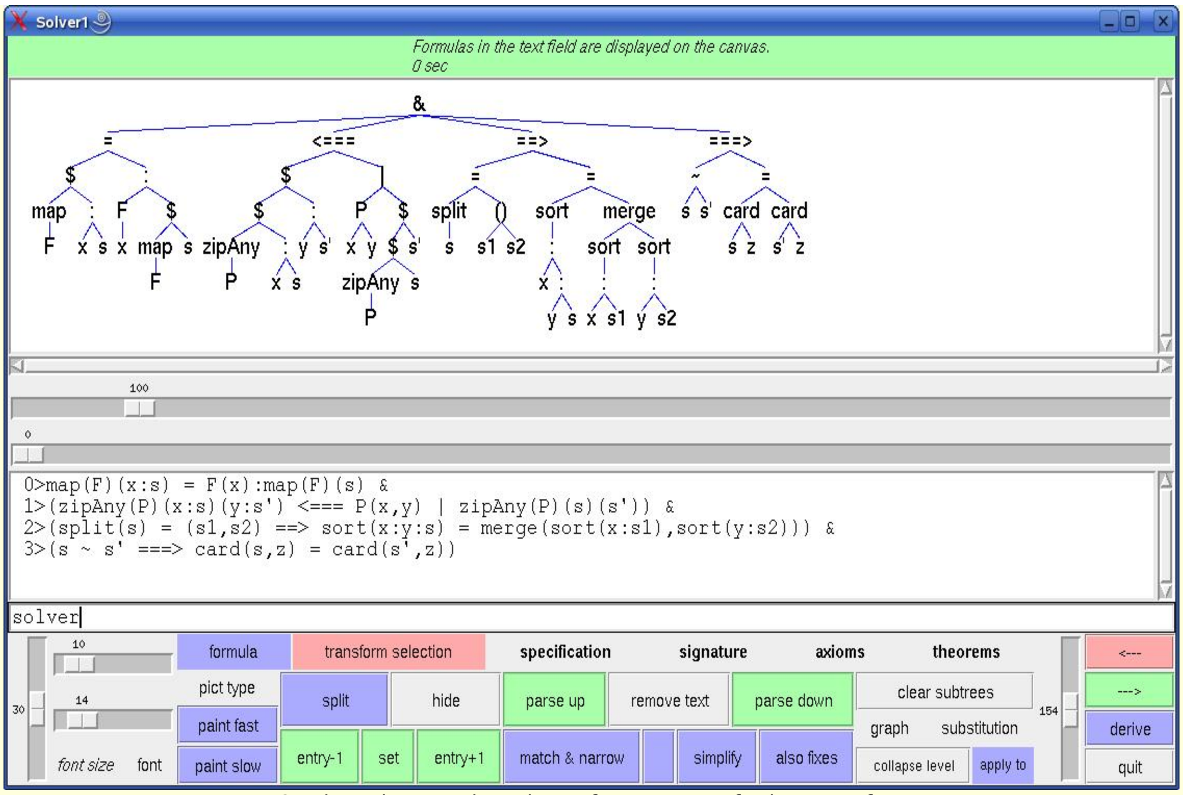
- ***Eterm*** contains data types and functions for generating, manipulating or checking terms and formulas, such as unification, matching, reduction and expansion of collapsed trees.

- ***Epaint*** provides Haskell functions for parsing terms and formulas and computing and displaying their graphical representations that are built up from Tk canvas widgets. Pictures can be defined as turtle movements over the plane (see *Widget interpreters*). The reactive components for animating the turtle and displaying graphical objects are part of the `painter`, `crawler` and `slowActor` templates (= classes). The `oscillator`template iterates a command with oscillating parameters. It is used for coloring error messages appearing in label fields and for animating dangling pointers.

- ***Esolve*** encapsulates translators between string, tree and graphical representations of terms and formulas. *Esolve* also contains the **simplifier** that partially evaluates terms and formulas. Moreover, the basic inference rules for applying axioms and theorems are implemented here. *Esolve* also contains the `enumerator` template that provides a GUI for running tree enumeration algorithms (see the sections *Alignments and palindromes* and *Dissections and partitions*). They are called from the `solver` template, which is part of *Ecom*.
- ***Ecom*** configures the GUI and provides all string- or tree-generating, -manipulating or -translating commands that the user may call for carrying out proofs or computations and presenting their results interactively. Multiple tree-shaped results can be displayed and browsed through on the canvas of a solver and in some cases interpreted graphically and displayed in the painter window of a solver (see the *paint buttons*). *Ecom* closes with the main program of the system that creates the main objects, partly in a mutually recursive way:

```
main tk = do
  win1 <- tk.window []
  win2 <- tk.window []
  fix solve1 <- solver tk "Solver1" win1 solve2 "Solver2" enum1 paint1
      solve2 <- solver tk "Solver2" win2 solve1 "Solver1" enum2 paint2
      paint1 <- painter tk solve1
      paint2 <- painter tk solve2
      enum1 <- enumerator tk solve1
      enum2 <- enumerator tk solve2
  solve1.buildSolve (0,20) solve1.buildSolveMore
  solve2.buildSolve (20,40) solve2.buildSolveMore
  win2.iconify
```

The solver, painter and enumerator templates make use of the O'Haskell module Tk.hs that provides the interface to Tcl/Tk (see the O'Hugs computing environments).


Formulas in the text field are displayed on the canvas.
0 sec

```
0>map(F)(x:s) = F(x):map(F)(s) &
1>(zipAny(P)(x:s)(y:s') <=== P(x,y) | zipAny(P)(s)(s')) &
2>(split(s) = (s1,s2) ==> sort(x:y:s) = merge(sort(x:s1),sort(y:s2))) &
3>(s ~ s' ===> card(s,z) = card(s',z))
```

```
solver
```

Fig. 2. *The solver window shows four axioms of a list specification.*

Solver features

Viewed from top to bottom, a solver window consists of the following widgets:

- a **label field** for displaying messages,
- a scrollable **canvas**,
- a horizontal **slider** for setting the number of tree nodes to be shown on the canvas,
- a horizontal **slider** for selecting the tree to be shown on the canvas,
- a scrollable and line-editable **text field**,
- an **entry field** for entering file names, node entries or integers,
- a vertical and a horizontal **slider** for stretching or osciing the tree horizontally resp. vertically,
- a horizontal **slider** for changing simultaneously the size of the canvas, text field, entry field and label field fonts,
- boldface-titled **menus** described below,
- framed **buttons** described below,
- a vertical **slider** for changing the relative vertical size of the canvas and the text field.

Solver state variables

- The current **signature** consists of symbols denoting
  - basic specifications (`specs`) consisting of signatures, axioms, theorems and/or conjectures,
  - predicates (`preds`) interpreted as the least solutions of their (Horn) axioms,
  - copredicates (`copreds`) interpreted as the greatest solutions of their (co-Horn) axioms,
  - constructors (`constructs`) for building up data,
  - defined functions (`defuncts`) specified by (Horn) axioms or implemented as Haskell functions called by the simplifier,
  - first-order variables (`fovars`) that may be instantiated by terms or formulas,
  - higher-order variables (`hovars`) that may be instantiated by functions or (co)predicates.

  For more details, see *Built-in signature*.
- The current **axioms** and **theorems** build up the high- or medium-level steps of a computation or proof. Axioms and theorems are applied to conjectures by rewriting or narrowing. A narrowing/rewriting step starts with unifying/matching a subtree (the redex) with/against an axiom. Narrowing applies (guarded) Horn or co-Horn clauses, rewriting applies only unconditional (guarded) equations. The guard of an axiom is a subformula to be solved before the axiom is applied. See also the *axioms menu*, *theorems menu*, *Axioms and theorems* and the *narrow/rewrite* buttons.
- The current **conjectures** is a list of arbitrary formulas derivable from the *Grammar*.
- **curr** holds the position of the actually displayed tree in the list of *current trees* (see below).
- **formula** indicates whether the list of current trees represents a disjunction or conjunction of formulas or a sum of terms, respectively. Conditional equations (see *Axioms and theorems*) applied to a formula should be valid in the initial model of the underlying swinging type, while conditional equations applied to a term may represent rewrite rules that are not valid equations. The results of the applications of several rewrite rules applied to the same term are combined with `<+>` to a set of terms (see *Built-in signature*).
- The Boolean variable **hideState** indicates whether selected subtrees are hidden or shown when the *hide/show* button is pressed. If no subtrees have been selected, pushing the *hide/show* button leads to a change of the value of *hideState*.
- The integer variable **maxHeap** yields the position of the maximal node w.r.t. the heap ordering up to which the current tree will be displayed. The value of *maxHeap* is set with the slider located directly below the canvas.
- The integer variable **matching** indicates the current strategy used for narrowing/rewriting (see the *narrow/rewrite* buttons).
- The Boolean variable **oneTree** indicates whether or not the current trees are split after narrowing, rewriting or simplification steps that are performed when no subtrees have been selected. If the *split/join* button is pushed, the current tree is split or joined, respectively, and the value of *oneTree* is changed.
- The widget interpreter **pictEval** recognizes paintable terms and transforms them into their pictorial representations. It is called via the *pict type menu*.
- The current **proof** records the sequence of derivation steps performed since the last initialization of the list of current trees (by *parsing* the contents of the text field; see *Derivations*) as a list of proof states each of which contains a description of the rule

that has been applied at last, the value of *treepos* before the rule has been applied, the resulting list of current trees and the resulting values of *treeMode*, *curr*, *varCounter*, *solPositions*, *fixPositions*, *substitution* and *subsDom*. One may browse among the proof states of the list by pushing the <--- or ---> button. If you push a button that triggers a proof step, the proof is continued in the state that you have switched to at last, i.e. subsequent states of the original proof are overwritten.

- The current **proof term** represents the current proof as an executable expression for the purpose of later proof checking. It is built up automatically in parallel to the construction of a derivation and can be saved to a user-defined file. A saved proof term is loaded by writing its name into the entry field and pushing *check proof term from file*. This action overwrites the current proof term. Starting out from the current tree, the proof represented by the loaded proof term is carried out stepwise by pushing the ---> button. Each click triggers a proof step and the proof term is entered into the text field with the constant `POINTER` preceding the command that will be executed next. If the entry field contains a positive natural number n, n proof steps are performed sequentially and only the final proof state is displayed. By pushing the <--- button one goes backwards. If the stop button is pushed, Expander2 leaves the proof check mode, i.e. the not-yet-evaluated part of the proof term is removed and all buttons regain their original function. Whenever the contents of the text field is parsed and thus turned into a new list of current trees, the proof term is initialized with commands that set the current values of *matching* (see above), *removeBit* and *simplifyBit* (see below).
- If the Boolean variable **removeBit** is set to `True`, a non-narrowable logical atom `P(t)` with normal form t is reduced to `False` if P is a predicate and to `True` if P is a copredicate. A non-rewritable term `f(t)` with normal form t is reduced to `()` (see *Built-in signature*). *This may lead to undesired effects if some function or (co)predicate has not been specified completely or if narrowing/rewriting is used in a match mode!* For instance, the simplifier turns a formula `Not(F)` into a negation-free formula that may contain unspecified complement predicates. Moreover, since `()` is a constructor, `() = ()` is simplified to `True`, while `() =/= ()` is simplified to `False`. Atoms of the form `() -> t` are also simplified to `False` (see the *narrow/rewrite buttons*).
- **rule** indicates whether the list of current trees is the result of narrowing steps, rewriting steps, simplification steps or other rule applications.
- The rules that may be applied in narrowing or rewriting steps either agree with the current axioms or are given by the clauses stored in the state variable **rules**. In the first case, *rules* is empty.
- The current **signature map** is a signature morphism from the current signature to the current signature of the other solver. It is initialized as the identity map on strings. Example (STACK2IMPL):

```
just -> entry
= -> ~
```

- If the Boolean variable **simplifyBit** is set to `True`, correct reducts of *apply clause*, *coinduction*, *fixpoint induction*, *instantiate*, *narrow/rewrite*, *remove* or *replace by other sides* is automatically simplified by at most 100 simplification steps (see the *simplify button*).
- The list **solPositions** consists of the positions of solved formulas resp. normal forms among the current trees.
- The pair **spread=(hor,ver)** yields the current horizontal resp. vertical space between adjacent nodes of some current tree (see below). *spread* is -- like the font of node labels -- also used by the painter associated with the solver.
- The current **substitution** maps the variables of its domain (= actual value of **subsDom**) to terms over the current signature. It is generated, modified and applied by particular buttons (see the *substitution menu* and the *apply to variable:* button).
- **treeMode** indicates whether the list **current trees** (= rooted graphs) is a singleton (treeMode = *tree*) or represents a disjunction of formulas (treeMode = *summand*), a conjunction of formulas (treeMode = *factor*) or a sum (= disjoint union) of terms (treeMode = *term*). `True`, `False` and `()` are the respective zero elements (see *Built-in signature*). The label of the *term/formula menu* shows the actual tree mode: If treeMode = *tree*", then the label is *term* resp. *formula*. If treeMode = *summand/factor*, then the label tells us how many summands resp. factors the set of current trees consists of. The slider between the canvas and the text field of a solver window allows one to browse among the current trees and to select the one to be displayed on the canvas. For the commands that may change *trees*, see the *term/formula menu*, the *transform-selection menu* and the *graph menu*.
- **treeposs** lists the positions of selected subtrees of the actually displayed tree. Subtrees are selected (and moved) by pushing the left mouse button while placing the cursor over their roots (see *Mouse and key events*).
- **varCounter** maps a variable x to the maximal index i such that xi occurs in the current proof. *varCounter* is updated when new variables are needed.

Built-in signature

```
preds:      -> <= >= < > >> _ all any zipAll zipAny `disjoint` `in` `NOTin` null `shares` `subset` `NOTsubset`
            Int Real reduced INV ~/~ ~/~0 ~/~1 ~/~2 ...
copreds:    ~ ~0 ~1 ~2 ...
constructs: <+> () [] ^ {} : 0 from_to bool cond fun lin inj0 inj1 inj2 ... suc
defuncts:   . ; + ++ - * ** / auto bag bisim blink concat count dnf drop foldl get0 get1 get2 ... head height
            init iter last length map mapt max `meet` min minimize `mod` nerode obdd parse permute postflow
            product reverse sat set shuffle stateflow subsflow sum tail take `then` tup upd0 upd1 upd2 ...
            zip zipWith
fovars:     i j x y z
```

`->` denotes a (labelled or unlabelled) transition relation (see below). `<=,>=,<,>` are predefined on integers, reals, strings and the defined functions `x_0,x_1,x_2,...` (used as node labels of OBDDs; see below). `` `subset` `` and `` `in` `` denote the subset resp. membership relation on all **collections**, i.e. terms `C(t1,...,tn)` where C is one of the constructors [] or {} or `C(t1,...,tn)=t1^...^tn`. `reduced` checks whether its argument is a variable-free tree without defined functions.

`Int(t)` and `Real(t)` return `True` if `t` is an integer or real number, respectively. `Int(t)` and `Real(t)` return `False` if `t` is a real or an integer number, respectively. `all(P)(as)` return `True` if all elements of `as` satisfy `P`. `any(P)(as)` return `True` if `as` contains an element that satisfies `P`. `zipAll(P)([a1,..,an])([b1,..,bn])` return `True` if for all 1 <= i <= n, (ai,bi) satisfies `P`. `zipAny(P)([a1,..,an])([b1,..,bn])` return `True` if for some 1 <= i <= n, (ai,bi) satisfies `P`.

If n > 1, then `length(t1,...,tn)` simplifies to n. `shuffle[ts]` shuffles the lists of ts before concatenating them. `height(t)` simplifies to the height of t, regardless of the semantics of t.

Formulas involving >> or `INV` are generated whenever an induction hypothesis or a (Hoare or subgoal) invariant is created (see the *transform-selection menu*). For the use of the underline symbol, see *enclose/replace by text*.

Terms combined with the infix constructor `<+>` are called **sum terms**. Semantically, `<+>` is a set union resp. insertion operator. The *simplifier* transforms a term of the form `f(...,t1<+>...<+>tn,...)` into the sum `f(...,t1,...)<+>...<+>f(...,tn,...)`.

`$` denotes the apply operator whose first argument is a higher-order term t that represents a predicate or function f. The other arguments of `$` are the arguments of f, i.e. `$(t,t1,...,tn)` stands for `t(t1,...,tn)`.

Given terms p1,...,pn,c and a formula u,

```
t = fun(p1,...,pn,bool(c)`then`u)
```

denotes a conditional λ-abstraction. The simplifier evaluates a corresponding application t(t1,...,tn) of t to (t1,...,tn) by matching (t1,...,tn) to (p1,...,pn), applying the unifier f to c and then to u provided that c[f] simplifies to True and (t1,...,tn) does not contain variables that are bound in t. Otherwise the simplification fails.

Given terms f1,...,fk, the application (f1;...;fk)(t1,...,tn) is simplified to fi(t1,...,tn) if i is the least m such that the simplification of fm(t1,...,tn) does nor fail.

tup(f1,...,fn)(t) is simplified to the list [f1(t),...,fn(t)]. Given a collector c, c(f1,...,fn)(t) is simplified to the collection c(f1(t),...,fn(t)).

A unary function f is applied repeatedly if a term of the form iter(f)(t) is simplified: n simplification steps transform this term into iter(f)(u) where u represents the value of f^n(t).

(), [] and {} denote tuple, list resp. set constructors of arbitrary finite arity. If () has no arguments, then () denotes "undefined" and is neutral with respect to the sum constructor <+>. A term of the form f((t1,...,tn)) is identified with f(t1,...,tn) provided that f is not a collector. Accordingly, for a variable x, f(x) unifies with f(t1,...,tn). The constructor : appends an element to a list from the left. 0 and suc are the natural number constructors. If applied to a number list s, suc returns the next permutation of s in reverse lexicographic order. In particular, if s is sorted, then suc(s)=reverse(s). The constructors inj0,inj1,inj2,... denote the first, second, third,... injection into a sum type. The simplifier decomposes (in)equations with the same leading constructor on both sides. The simplifier also replaces (in)equations with different leading constructors on both sides by False (True).

bool, cond and lin embed formulas into terms (see the *Grammar*).

+,-,*,**,/,`mod`,max,min are defined on integer and real numbers. +,-,*,/ work also for polynomials (* and / only as scalar operators).

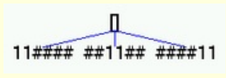
Given finite lists or sets s,s' and integers i,k, s-s' and [i..k] denote the list of elements of s that are not in s' and the interval of integers from i to k, respectively. from_to is an internal constructor. For instance, the parser translates the string [t..u] to the term from_to(t,u), while the simplifier compiles from_to(t,u) to the corresponding interval if t and u have been evaluated to integers. Haskell shortcuts like [i,k..n] may also be used. ., ++, concat, drop, foldl, head, init, last, length, map, null, product, sum, tail, take, reverse, zip and zipWith are defined as the synonymous Haskell functions. Some functions on lists also apply to bags and sets.

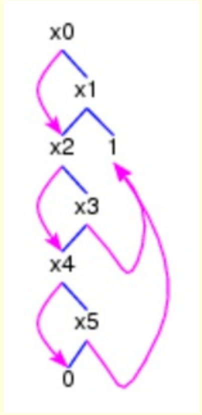
any, all, map, foldl, zip, zipAny zipAll and zipWith also occur in LIST and LISTEVAL with (recursive) axioms. The synonymous built-in symbols are interpreted as partial non-recursive functions. For instance, a rewriting step via LIST transforms the term map(suc)(x:s) into x:map(suc)(s), while the simplifier does not modify this term, but would turn map(suc)[x,y,z] into [suc(x),suc(y),suc(z)]. Of course, axioms introduced for built-in symbols should comply with their built-in interpretation that is realized by the simplifier.

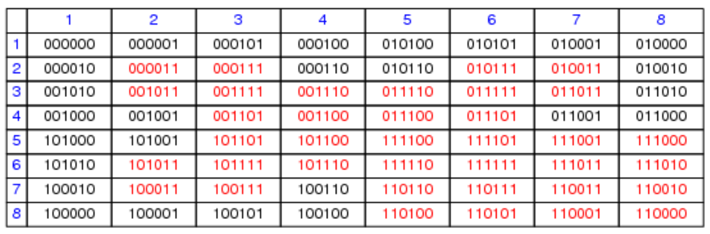
count(ts,t) counts the number of occurrences of t in the list ts. ts `disjoint` us checks whether the lists ts and us are disjoint. ts `meet` us computes the intersection of the lists ts and us. ts `shares` us checks whether the lists ts and us are not disjoint.

get0,get1,get2,... and upd0,upd1,upd2,... return resp. update the first, second, third,... component of a tuple or element of a collection.

bag transforms a list into a bag and flattens terms built up with the infix operator ^ (see below). set turns a list or bag into a set. Many functions defined on lists are also defined on other collections. For obvious semantical reasons, the simplifier applies count, `disjoint`, `in`, `meet`, `shares`, ++, - and <= only to variable-free terms without defined functions.







	1	2	3	4	5	6	7	8
1	000000	000001	000101	000100	010100	010101	010001	010000
2	000010	000011	000111	000110	010110	010111	010011	010010
3	001010	001011	001111	001110	011110	011111	011011	011010
4	001000	001001	001101	001100	011100	011101	011001	011000
5	101000	101001	101101	101100	111100	111101	111001	111000
6	101010	101011	101111	101110	111110	111111	111011	111010
7	100010	100011	100111	100110	110110	110111	110011	110010
8	100000	100001	100101	100100	110100	110101	110001	110000

Fig. 3. *A DNF (DNF5), its minimal OBDD and its Karnaugh diagram*

obdd transforms a DNF represented as a list of strings of the same positive length whose characters are 0, 1 or # into an equivalent minimal OBDD. dnf transforms OBDDs into equivalent minimal DNFs. If applied to a DNF, minimize minimizes the number of summands of a DNF. If applied to an OBDD, minimize minimizes the number of nodes of an OBDD according to the two reduction rules for OBDDs [Bry].

^ is an infix operator for building bags and treated by the unification algorithm as an associative and commutative function. When a **bag term** t1^...^tn in the displayed tree is to be unified with another bag term u, then the unification succeeds even if only a permutation of t1^...^tn unifies with u. If there are several unifiers, those are preferred, which substitute only variables for variables. Among these unifiers those are preferred, which substitute variables only for variables of u.

Axioms of the form

```
          {guard ==>} (t1^...^tn -> u {<=== prem})           (*)
```

are called **transitional axioms**. (*) can be applied to a bag term t = `u1^...^um` if the list `[t1,...,tn]` unifies with a list `[ui1,...,uin]` of elements of t such that 1<=i1<... <=m, the unifier f satisfies `guard` and t is the left-hand side of a **transitional atom** `t -> t'`. This atom is then replaced by the instance of the formula

```
          (u^uk1^...^uk(m-n) = t' {& prem}
```

by f. If u = `[v1,...,vk]` for some k>1, then (*) is treated as a conjunction of the clauses

```
          {guard ==>} (t1^...^tn -> v1 {<=== prem})
          ...
          {guard ==>} (t1^...^tn -> vk {<=== prem})
```

Set brackets used in clauses enclose optional subformulas, i.e. `guard` and `prem` in axiom `(*)` may be empty.

If the application of (*) to t fails, the elements of t are permuted. If after 100 permutations (*) is still inapplicable, the last permutation of t will be returned as result - and yield a new starting point for further attempts to apply (*).

For instance, repeated applications of the AC rule

```
          i`mod`j = 0 ==> i^j -> j
```

(see PRIMS) to 2^3^4^5^6^7^8^9^10^11^12^13^14^15 sift out the primes and thus end up with 2^3^5^7^11^13. ACCOUNT, BOTTLEAC, PUZZLE and the *algebraic net specifications* PHILAC and ECHOAC also contain AC rules.

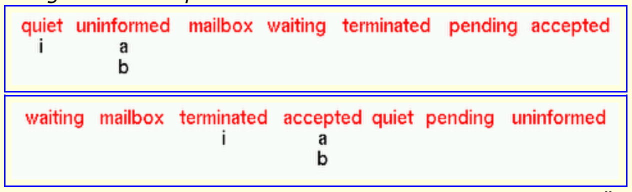


Fig. 4. *Snapshots of a run of the echo algorithm (cf. [SMÖ])*

The symbols `&,|,=,=/=,+,*,^,{},<+>,~,~0,~1,~2,...,~/~,~/~0,~/~1,~/~2,...` are **permutators**, i.e. the order of their arguments is irrelevant. Consequently, AC unification replaces ordinary unification whenever the respective arguments of a permutator are to be unified.

The symbols `^,{},++` and `<+>` are treated as associative operators and thus may have an arbitrary finite number of arguments. `{}` and `<+>` are idempotent. `[]` is neutral with respect to `^,{}` and `++`. `()` is neutral with respect to `<+>`.

`Actions,Atoms,Finals,FinalsL,Fix,Matrix,MatrixL,Perm,Trans` and `TransL` denote state terms to be initialized by equational axioms and modified by *simplifications* (see also the *narrow* button).

`~,~0,~1,~2,...` are declared as copredicates and used as congruence relations. They are supposed to denote *behavioral* equalities. `<,>,=/=,~/~,~/~0,~/~1,~/~2,...` are the complements of `>=,<=,=,~,~0,~1,~2,...`, respectively. The simplifier replaces behavioral (in)equations with different leading injections or tuples of different length on both sides by `False` (`True`).

For each other predicate or copredicate `P`, `notP` denotes the complement of `P`. Axioms for the complement of `P` are added to the current axioms if `P` is entered into the entry field and the button *negate axioms for symbol* is pushed.

Subformulas involving built-in functions or predicates are (partially) evaluated when the displayed tree is simplified. This includes the stepwise execution of built-in functions with state term parameters (see *Simplifications*).

The declaration of a copredicate p overwrites preceding declarations of p as a predicate or constructor. The declaration of a predicate or defined function f overwrites preceding declarations of f as a constructor. The declaration of a defined function x overwrites preceding declarations of x as a first-order variable (like in OBDD).

For example, the signature OBDD reads as follows:

```
     defuncts: restrict forall exists quantor x X Y F and or not
     fovars:   u2 u1 u t2 t1 t j i b
     hovars:   F{and,or} X{x} Y{x}
```

`F{and,or}` denotes that the defined functions `and` and `or` are the only admissable instances of the higher-order variable `F`. In general, the list of strings following a higher-order variable `F` consists of symbols that are **admissable for** `F`. In addition to the symbols of the list, all higher-order variables are admissable for `F`. If `F` is not followed by a list of strings, then all symbols of the current signature are admissable for `F`.

Keywords (`specs:`, `preds:`, `copreds:`, `constructs:`, `defuncts:`, `fovars:` and `hovars:`) may appear at any place in the list of symbols that builds up a signature. To be recognized as keywords they must be separated from their context by blanks. User-defined signatures automatically inherit the built-in signature. Symbols that are to be interpreted as infix operators must start and end with the character ` or consist of characters among

```
     : + - * < = ~ > / ^ #
```

(see the language generated by infixToken in the *Grammar*). Symbols used in axioms, theorems or conjectures that do not belong to the current signature are interpreted as (undefined) function symbols (see the *Grammar*). This facilitates certain applications, but may also lead to unexpected unification failures when axioms or theorems are applied.

Mouse and key events

A subtree t is selected (or deselected if it has already been selected) by clicking on the **left mouse button** while placing the cursor over its root. If the mouse is moved while the button is pressed, t is shifted over the canvas. If the button is released while the root of t is placed over the root of another subtree u, u is replaced by t. If u is an existentially (resp. universally) quantified variable and the scope of u has positive (resp. negative) polarity, then all occurrences of u within the scope are replaced by t. Hence, in these cases, the replacement works like *instantiate*.

Subtrees are deselected backwards with respect to the order in which they were selected by moving the cursor away from the displayed tree and pushing the left mouse button. All selected subtrees are deselected simultaneously if the *clear subtrees* button is pushed. If you stop moving a subtree before inserting it into the displayed tree, it will stay at the place where you released the mouse button. Then push the *redraw* button and the subtree will be returned to its previous place within the displayed tree.

If a subtree t has been selected and the move of t is started with a click on the **middle mouse button**, t will be removed from the displayed tree and replaced by the variable **zn** where the index n is increased each time a subtree is removed or a new variable is needed when an axiom is flattened (see below). Moreover, the current substitution is extended by the assignment of t to **zn**.

By moving the mouse and pushing the middle button outside the root of a selected subtree the entire displayed tree is shifted over the canvas. By pressing the **right mouse button** while placing the cursor over a node x a **pointer** (edge) from x to the root of the last selected subtree t is drawn and all successors of x are removed. The arc is orange-colored if it closes a circle consisting of edges of t. Otherwise it is magenta-colored. Subtree replacements and substititutions for variables adapt the pointer values.

A command followed by a letter in round brackets is executed when the key with the letter is pressed after the cursor has been placed over the label field and the left mouse button has been pushed (see *Commands*).

term/formula menu

The commands of the term/formula menu create or transform the current trees or the current proof.

- ***call enumerator*** opens a submenu listing tree enumeration algorithms. When you push the button for one of these algorithms, you will be prompted to enter sequences of strings (in the case of the alignment or palindrome enumerator), numbers (in the case of the dissection enumerator) or the length of a list (in the case of the partition enumerator) and certain constraints (see the sections *Alignments and palindromes* and *Dissections and partitions*). After the "go" button has been pushed, the resulting trees are assigned to Solver1/2 and may be browsed through with the canvas slider.
- ***remove other trees*** eliminates all current trees except the current one.
- ***show changed*** selects all maximal elements within the set of subtrees that have been modified during the last transformation of the displayed tree.
- ***show proof*** enters the current proof into the text field.
- ***.. in text field of Solver1/2*** opens Solver1/2 and enters the current proof into its text field.
- ***save proof to file (p)*** saves the current proof to *Examples/file* if *file* is the string in the entry field.
- ***show proof term*** enters the current proof term into the text field.
- ***.. in text field of Solver1/2*** opens Solver1/2 and enters the current proof term into its text field.
- ***save proof term to file (t)*** saves the current proof term to *Examples/file* if *file* is the string in the entry field.
- ***check proof term in file (c)*** assigns the contents of the file in the entry field to the current proof term provided that the contents is a proof term.
- ***.. text field*** assigns the contents of the text field to the current proof term provided that the contents is a proof term.
- ***create induction hypotheses*** prepares the displayed tree cl for a proof by Noetherian induction. The command assumes that cl is a formula and that free or universal *induction* variables `x1,...,xn` of cl have been selected, which will be prefixed by an exclamation mark. Non-selected free variables are turned into universal ones. If cl has the form `prem ==> conc`, then the clauses

  ```
  conc' <=== (x1,...,xn) >> (!x1,...,!xn) & prem'
  prem' ===> ((x1,...,xn) >> (!x1,...,!xn) ==> conc')
  ```

  are added to the current theorems. The primed formulas are obtained from the unprimed ones by replacing `xi` with `!xi`, 1 <= i <= n. If cl is not an implication, then

  ```
  cl' <=== (x1,...,xn) >> (!x1,...,!xn)
  ```

  is added to the current theorems.
- ***flatten (co-)Horn clause*** assumes that the displayed tree is a Horn or co-Horn clause cl (see *Axioms and theorems*). If subterms t1,...,tn of cl are selected and F is the set of roots of t1,...,tn, then cl is replaced by an equivalent formula where each f ∈ F occurs only at the outermost position of the left- or right-hand side of an equation. If no subterms are selected, F is the set of all defined functions of the current signature. For instance, the LISTEVAL-axiom

  ```
  sort(x:(y:s)) = merge(sort(x:s1),sort(y:s2)) <=== split(s) = (s1,s2)
  ```
  is turned into
  ```
  sort(x:(y:s)) = merge(z0,z1) <=== split(s) = (s1,s2) & sort(x:s1) = z0 & sort(x:s2) = z1
  ```
  if F = {sort} and into
  ```
  sort(x:(y:s)) = z0 <=== split(s) = (s1,s2) & merge(z1,z2) = z0 & sort(x:s1) = z1 & sort(y:s2) = z2
  ```
  if F = {sort,merge}.
- ***turn local def into function application*** takes the last local definition (= equation with a normal form p on the right-hand side) in the displayed tree, which is supposed to be a conditional equation, and transforms it into an equivalent function application. For instance,
  `l=r <=== prem & t=p` becomes `l=fun(p,r)(t) <=== prem.`
- ***save tree to file*** saves the string representation of the displayed tree to *Examples/file* if *file* is the string in the entry field.
- ***save tree in eps format to file (i)*** saves the current tree in Encapsulated PostScript format to *Pics/file* if *file* is the string in the entry field.
- ***save trees to file*** saves the string representation of the disjunction, conjunction or sum, respectively, of the current trees to *Examples/file* if *file* is the string in the entry field.
- ***load text*** opens a submenu of files. The contents of the selected file is entered into the text field.

font menu

consists of buttons for choosing the font to be used for the text in tree nodes and pictorial term representations. The font size is controlled by a slider (see *Solver features*).

transform-selection menu

The commands of this menu transform the subtrees that were selected with the left mouse button. If no subtree has been selected, the entire displayed tree is regarded as being selected. Most commands call inference rules and deliver messages that tell us whether or not the executed rule application is sound with respect to the initial model induced by the current signature and axioms (see *Derivations*).

- ***copy*** adds a copy of the subtree selected at last to the children of its parent node.
- ***remove*** removes all selected subtrees if they are summands/factors of the same disjunction/conjunction with positive/negative

polarity. Otherwise the greatest lower bound of the selected subtrees is removed.

- ***reverse (r)*** reverses the list of at least two selected subtrees. The reduct implies the redex if the subtrees have the same direct predecessor x and if x is a *permutator* (see *Built-in signature*). If only one subtree t is selected, then the operation is applied to the list of maximal proper subtrees of t.
- ***enclose/replace by text (e)*** assumes the selection of subtrees t1,...,tm. Moreover, the text field is supposed to contain a tree u with n leaves labelled with a wildcard symbol (_). If n=0, then u is substituted for t1,...,tm. If n=1 and t1,...,tm are orthogonal to each other or if n>1 and m=1, then for all 1<=i<=m, u[ti/_] is substituted for ti. Otherwise t1 is supposed to enclose t2,...,tm and m-1 is supposed to be equal to n. Then t1 is replaced by the term obtained from u by replacing the n leaves of u labelled with _ by t2,...,tm, respectively.
- ***instantiate*** assumes the selection of a quantified variable x. If x is existential resp. universal and the scope of x has positive resp. negative polarity (see *Derivations*), then all occurrences of x are replaced by the term in the entry field. See also *Mouse and key events*.
- ***unify*** assumes the selection of two factors or summands t and u of a conjunction resp. disjunction. If t and u are unifiable and the unifier instantiates only existential resp. universal variables of the conjunction resp. disjunction, then t is removed and the unifier is applied to the remaining conjunction resp. disjunction.
- ***generalize*** combines the last selected subformula `F` of the displayed tree with the formula `G` in the entry field. If `F` has positive polarity, then `F&G` replaces `F`. Otherwise `F|G` replaces `F`. A generalization of `F` may be necessary before `F` can be proved by Noetherian induction, fixpoint induction or coinduction.
- ***decompose atom*** assumes the selection of an atom `t=t'`, `t~t'` or `t~k t'` for some natural number k with positive polarity or an atom `t=/=t'`, `t~/~t'` or `t~/~k t'` for some natural number k with negative polarity. The selected atom is decomposed in accordance with the assumption that =, ~ and ~k are compatible with all function symbols (see *Built-in signature*).
- ***replace by other sides of equations*** assumes that the subtree t selected at first is an implication/conjunction and the other selected subtrees t1,...,tn are subterms of t. For all 1 <= i <= n, the command searches for an atom *ti=ui* or *ui=ti* in the premise or among the other factors of t, respectively. For all 1 <= i <= n, ti is replaced by ui. The replacement of t is correct whenever = is compatible with all function and relation symbols (see *Built-in signature*).
- ***.. of inequations*** assumes that the subtree t selected at first is an implication/disjunction and the other selected subtrees t1,...,tn are subterms of t. For all 1 <= i <= n, the command searches for an atom *ti=/=ui* or *ui=/=ti* in the conclusion or among the other summands of t, respectively. For all 1 <= i <= n, ti is replaced by ui. The replacement of t is correct whenever = is compatible with function and relation symbols (see *Built-in signature*).
- ***use transitivity*** assumes the selection of an atom `t R t'` with positive polarity or factors `t1 R t2, t2 R t3, ..., t(n-1) R tn` of a conjunction with negative polarity (see *Derivations*) such that R is among `=, ~, ~k, <=, >=, <, >`. The selected atoms are decomposed resp. composed in accordance with the assumption that R is transitive (see *Built-in signature*).
- ***apply clause in entry field*** applies the n-th clause cl in the text field to all selected subtrees provided that the current trees are formulas and the entry field contains the number n. cl may be applied from left to right or from right to left where left/right refers to t resp. u if cl has the form `tRu <=== prem` where R is symmetric and to the formula left/right of `<===` resp. `===>` in all other cases. If cl is distributed, then cl's atoms must unify componentwise with the selected subtrees. Otherwise cl is applied to each selected subtree (see *Axioms and theorems*).
- ***.. in text field*** applies the clause in the text field analogously to the previous command.
- ***.. and save redex*** adds the redex disjunctively/conjunctively to the reduct if the clause is a non-distributed Horn/co-Horn clause. The correctness of this version of the rule does not depend on the polarity of the redex.
- ***move up quantifiers*** assumes the selection of quantified arguments of a propositional operator op, i.e. op in {&, |, `Not` or ==>}. The quantifiers are shifted in front of op after all bound variables that also occur freely in some argument or in more than one argument of op have been renamed. For instance, a distributed clause of type (9) or (11) cannot be applied to existentially quantified factors and a clause of distributed type (8) or (10) cannot be applied to universally quantified summands (see *Axioms and theorems*). Hence moving the quantifiers out of the conjunction resp. disjunction may be necessary.
- ***shift subformulas*** shifts all selected factors of the premise and all selected summands of the conclusion of an implication `prem==>conc` to `conc` and `prem`, respectively. Such a transformation may be necessary if `prem==>conc` shall be proved by fixpoint induction or coinduction.

For each predicate, copredicate or function p, let AX(p) be the set of axioms for p.

- ***coinduction*** assumes the selection of conjectures

  ```
    {prem1 ==>} p(t11,..,t1n)
  & ...                                                    (A)
  & {premk ==>} p(tk1,..,tkn)
  ```

  about a copredicate p that does not depend on any predicate or function occurring in premi. A is *stretched* into

  ```
  p(x1,...,xn) <===   {prem1 &} x1=t11 & ... & xn=t1n
                    | ...                                  (A')
                    | {premk &} x1=tk1 & ... & xn=tkn
  ```

  x1,...,xn are variables. In fact, only those terms among ti1,..,tin are replaced by variables that are not variables or occur more than once among ti1,..,tin. Morever, a new predicate p' is added to the current signature and

  ```
  p'(x1,...,xn) <===   {prem1 &} x1=t11 & ... & xn=t1n
                     | ...                                 (AX0)
                     | {premk &} x1=tk1 & ... & xn=tkn
  ```

  become the axioms for p'.

  Let m be the number in the entry field (default: m=0). For all p(t)===>F in AX(p), let G be the result of submitting F to a sequence of m inference steps each of which consists of the parallel application of AX(p) to all current redices. AX0 is applied to p'(t)==>G[p'/p]. The conjunction of the resulting clauses replaces the original conjecture A.

  k>1 conjectures of the form A may be selected, which are assumed to be factors of the same conjunction and to deal with different copredicates p1,...,pk. Then the above described transformation is applied to the set of k selected conjectures.

- ***.. and save redex*** works the same as the preceding command except that each at===>F in AX(p) is replaced by at===>F|at.
- ***fixpoint induction*** assumes the selection of conjectures

  ```
    p(t11,..,t1n) ==> conc1
  & ...                                                    (B)
  & p(tk1,..,tkn) ==> conck
  ```

  about a predicate p that does not depend on any predicate or function occurring in conci or a conjecture of the form

```
                   f(t11,..,t1n) = t1 ==> conc1
                 & ...                                                                (C)
                 & f(tk1,..,tkn) = tk ==> conck
```

or

```
                   f(t11,..,t1n) = t1 {& conc1}
                 & ...                                                                (D)
                 & f(tk1,..,tkn) = tk {& conck}
```

about a defined function f that does not depend on any predicate or function occurring in `ti` or `conci`. (B), (C) and (D) are *stretched* into

```
                   p(x1,...,xn) ===> (x1=t11 & ... & xn=t1n ==> conc1)
                 & ...                                                                (B')
                 & (x1=tk1 & ... & xn=tkn ==> conck),

                   f(x1,...,xn) = x ===> (x1=t11 & ... & xn=t1n & x=t1 ==> conc1)
                 & ...                                                                (C')
                 & (x1=tk1 & ... & xn=tkn & x=tk ==> conck)
```

and

```
                   f(x1,...,xn) = x ===> (x1=t11 & ... & xn=t1n ==> x=t1 {& conc1})
                 & ...                                                                (D')
                 & (x1=tk1 & ... & xn=tkn ==> x=tk {& conck}),
```

respectively. `x1,...,xn,x` are variables. In fact, only those terms among ti1,..,tin are replaced by variables that are not variables or occur more than once among ti1,..,tin. Morever, a new predicate p' resp. f' is added to the current signature and

```
                   p'(x1,...,xn) ===> (x1=t11 & ... & xn=t1n ==> conc1)
                 & ...                                                    (AX0)
                 & (x1=tk1 & ... & xn=tkn ==> conck)
```

resp.

```
                   f'(x1,...,xn,x) ===> (x1=t11 & ... & xn=t1n & x=t1 ==> conc1)
                 & ...                                                    (AX0)
                 & (x1=tk1 & ... & xn=tkn & x=tk ==> conck)
```

resp.

```
                   f'(x1,...,xn,x) ===> (x1=t11 & ... & xn=t1n ==> x=t1 {& conc1})
                 & ...                                                    (AX0)
                 & (x1=tk1 & ... & xn=tkn ==> x=tk {& conck})
```

become the axioms for p' resp. f'.

Let m be the number in the entry field (default: m=0). For all p(t)<===F ∈ AX(p) resp. f(t)=u<===F ∈ flat(AX(f)), let G be the result of submitting F to a sequence of m inference steps each of which consists of the parallel application of AX(p) resp. flat(AX(f)) to all current redices. AX0 is applied to G[p'/p]==>p'(t) resp. G[f'/(f(-)=-)]==>f'(t,u). The conjunction of the resulting clauses replaces the original conjecture B/C/D.

k>1 conjectures of the form B/C/D may be selected, which are assumed to be factors of the same conjunction and to deal with different predicates p1,...,pk resp. functions f1,...,fk. Then the above described transformation is applied to the set of k selected conjectures.

- ***.. and save redex*** works the same as the preceding command except that each at<===F ∈ AX(p) resp. at<===F ∈ AX(f) is replaced by at<===F&at.

- ***create Hoare invariant*** assumes the selection of a conjecture of the form

```
                 f(t1,..,tn) = t ==> conc          (A)
```

or

```
                 f(t1,..,tn) = t {& conc}          (B)
```

such that f is a **derived function**, i.e. f has a single axiom of the form

```
                 f(x1,...,xn) = g(u1,...,uk)
```

or, if ti in A/B has been selected (in addition to A/B itself), f has a single axiom of the form

```
                 f(x1,...,xn) = g(xi,...,xn,u1,...,uk)
```

with distinct variables x1,...,xn. Let

```
                 INV(x1,...,xn,u1,...,uk)                                (INV1)
                 g(xi,...,xn,y1,...,yk) = z & INV(x1,...,xn,y1,...,yk)
                               ==> (x1=t1 & ... & xn=tn & x=t ==> conc)   (INV2)
                 g(xi,...,xn,y1,...,yk) = z & INV(x1,...,xn,y1,...,yk)
                               ==> (x1=t1 & ... & xn=tn ==> x=t {& conc}) (INV3)
```

A is turned into INV1 & INV2, while B is turned into INV1 & INV3. If ti has not been selected in A/B, then g(xi,...,xn,y1,...,yk) reduces to g(y1,...,yk). Usually, the proof proceeds by narrowing INV1, shifting INV(x1,...,xn,y1,...,yk) from the premise to the conclusion of INV2/INV3 and submitting the resulting formula to fixpoint induction.
- ***create subgoal invariant*** works the same as the preceding command except that a selected conjecture of the form A or B is turned into INV1 & INV2 and INV1 & INV3, respectively,where

```
                 INV(xi,...,xn,u1,...,uk,z) ==> (x1=t1 & ... & xn=tn & x=t ==> conc) (INV1)
                 g(xi,...,xn,y1,...,yk) = z ==> INV(xi,...,xn,y1,...,yk,z)           (INV2)
                 g(xi,...,xn,y1,...,yk) = z ==> INV(xi,...,xn,y1,...,yk,z)           (INV3)
```

Usually, the proof proceeds by narrowing INV1 and submitting INV2/INV3 to fixpoint induction.
- ***replace by tree of Solver1/2*** replaces the subtree selected at last by the displayed tree of Solver1/2.
- ***unify with tree of Solver1/2*** unifies the subtree selected at last with the displayed tree of Solver1/2.
- ***build unifier*** assumes the selection of two subtrees. If they are unifiable, the most general unifier is assigned to the current substitution. Otherwise the reason for the failure is reported.
- ***subsume*** assumes the selection of the premise t and the conclusion u of an implication or two factors t and u of a conjunction

or two summands t and u of a disjunction. If t subsumes u, then t==>u is replaced by True or u is removed from the conjunction or t is removed from the disjunction, respectively.
- ***stretch premise*** assumes the selection of a formula of the form B/C/D (see above). The formula is turned into the corresponding co-Horn clause of the form B'/C'/D'.
- ***stretch conclusion*** assumes the selection of a formula of the form A (see above). The formula is turned into the corresponding Horn clause of the form A'.

specification menu

- ***re-add*** removes the current specification and re-adds the specification contained in the file that has been added at last (and possibly modified in the meantime).
- ***remove*** removes the current specification except for the *built-in symbols*. The current signature map is set to the identity map and the widget interpreter *pictEval* to *matrix* (see the *pict type menu*).
- ***save to file*** saves the current specification to *Examples/file* if *file* is the string in the entry field.
- ***load text*** opens a submenu of files. The contents of the selected file is entered into the text field.
- ***add*** opens a submenu of specification files. The files may contain signature elements, axioms, theorems and/or conjectures (in this order!). Signature elements, axioms, theorems are added to the current signature, axioms, theorems and conjectures, respectively. Then all conjectures are entered into the text field. Axioms, theorems and conjectures must be preceded by the keywords `axioms:`, `theorems:` and `conjects:`, respectively. Axioms must be given as a conjunction of guarded Horn or co-Horn clauses. Theorems must be given as a conjunction of Horn or co-Horn clauses (see *Axioms and theorems* and the *narrow* button). Conjectures may be given as sums of terms or conjunctions of formulas.

signature menu

- ***remove map*** reduces the current signature map to the identity.
- ***show sig*** enters the current signature into the text field.
- ***show map*** enters the current signature map into the text field.
- ***apply map*** applies the current signature map to the current tree and displays the result in the other solver.
- ***save map to file*** saves the current signature map to *Examples/file* if *file* is the string in the entry field.
- ***add map*** opens a submenu of files whose contents is compiled into an extension of the current signature map when the respective menu button is pushed.

axioms menu

- ***remove axioms*** removes the current axioms.
- ***.. in entry field*** removes the n-th clause in the text field from the current axioms provided that the entry filed contains the number n.
- ***.. for symbols*** removes the axioms for the roots of the subtrees selected at last (or the symbols in the entry field if no subtrees have been selected) from the current axioms.
- ***remove rules*** empties the state variable *rules*.
- ***set rules to clauses in entry field*** assumes that the entry field contains a list of numbers of clauses in the text field. These clauses are assigned to *rules*.
- ***.. to axioms for symbols*** assigns to i>rules the axioms for the roots of the selected subtrees (or for the symbols in the entry field if no subtrees have been selected).
- ***negate for symbols*** adds axioms for the complements of the roots of the subtrees selected at last (or the (co)predicates in the entry field if no subtrees have been selected) to the current axioms. For instance, Horn axioms for `sorted` read as follows (see LIST):

```
sorted[]
sorted[x]
sorted(x:(y:s)) <=== x <= y & sorted(y:s)
```

  negate for symbols transforms them into the following co-Horn axioms for `NOTsorted`:

```
NOTsorted[] ===> False
NOTsorted[x] ===> False
x <= y ==> (NOTsorted(x:(y:s)) ===> NOTsorted(y:s))
```

- ***invert for symbols*** transforms the axioms for the roots of the subtrees selected at last (or the (co)predicates in the entry field if no subtrees have been selected) into a single (co-)Horn clause that represents the inverse of the axioms. The clause expresses the *least/greatest* fixpoint semantics of the predicates and is thus added to the current theorems. For instance, *invert for symbols* turns the above Horn axioms for `sorted` into the following co-Horn theorem:

```
sorted(z) ===> z = [] |
               Any x: z = [x] |
               Any x y s: (z = x:(y:s) & x <= y & sorted(y:s))
```

  The clause resulting from inverted axioms is indeed a theorem with respect to the underlying least/greatest fixpoint semantics of predicates/copredicates.
- ***Kleene axioms for symbols*** transforms the axioms for the roots of the subtrees selected at last (or for the symbols in the entry field if no subtrees have been selected) into equivalent (co-)Horn axioms (see [P03]). For instance, the above co-Horn axioms for `NOTsorted` are turned into (equivalent) Horn axioms:

```
NOTsorted(z) <=== All i: NOTsortedLoop(i,z)
NOTsortedLoop(0,z)
NOTsortedLoop(suc(i),z) <=== z =/= [] &
                             All x: z =/= [x] &
                             All x y s: (z = x:(y:s) & x <= y ==> NOTsortedLoop(i,y:s))
```

  The above Horn axioms for `sorted` are turned into (equivalent) coHorn axioms:

```
sorted(z) ===> Any i: sortedLoop(i,z)
sortedLoop(0,z) ===> False
sortedLoop(suc(i),z) ===> z = [] |
Any x: z =/= [x] &
Any x y s: (z = x:(y:s) & x <= y & sortedLoop(i,y:s))
```

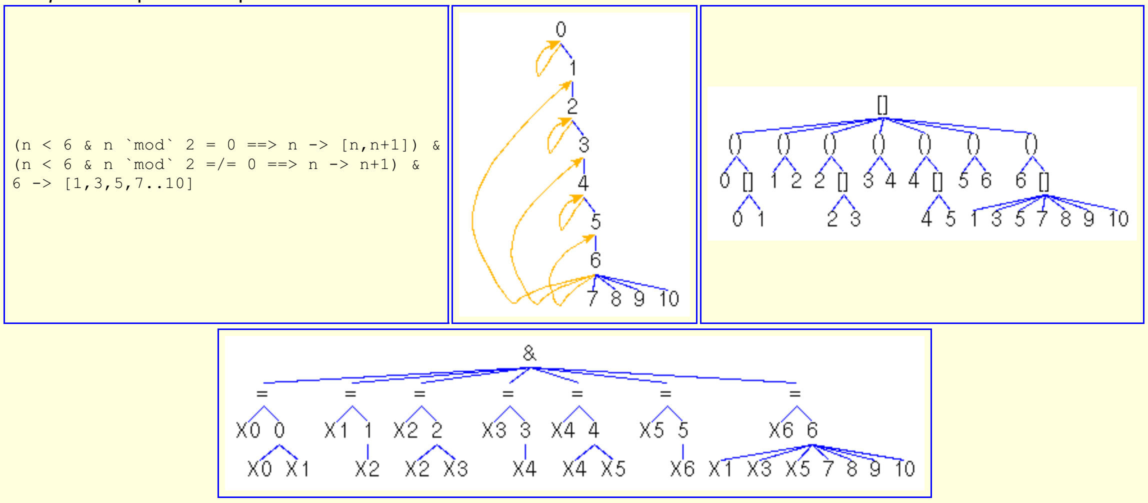
The old axioms for the symbols are deleted. Copredicates become predicates. Predicates become copredicates. Axioms for functions are not handled.
- ***show axioms*** enters all current axioms into the text field.
- ***.. for symbols*** enters into the text field the axioms for the roots of the selected subtrees (or for the symbols in the entry field if no subtrees have been selected).
- ***.. in text field of Solver1/2*** enters all current axioms of Solver 1/2 into the text field of Solver1/2.
- ***add*** opens a submenu of files, each one containing a disjunction of at most two conjunctions of guarded Horn or co-Horn clauses (see *Axioms and theorems*). The factors of the conjunctions are added to the current axioms. Signature elements declared in the file are added to the current signature. In this case, the factors must be preceded by the keyword `axioms:`.

theorems menu

- ***remove theorems*** removes the current theorems.
- ***.. in entry field*** removes the n-th clause in the text field from the current theorems provided that the entry filed contains the number n.
- ***remove conjects*** removes the current conjectures.
- ***.. in entry field*** removes the n-th term or formula in the text field from the current conjectures provided that the entry field contains the number n.
- ***show theorems*** enters all current theorems into the text field.
- ***... for symbols*** enters the theorems for the roots of the subtrees selected at last (or the symbols in the entry field if no subtrees have been selected) into the text field.
- ***... in text field of Solver1/2*** enters all current theorems of Solver 1/2 into the text field of Solver1/2.
- ***show conjects*** enters all current conjectures into the text field.
- ***save to file*** saves the current theorems to *Examples/file* if *file* is the string in the entry field.
- ***add theorems*** opens a submenu of files, each one containing a conjunction of Horn or co-Horn clauses (see *Axioms and theorems*). The factors of the conjunction are added to the current theorems. Signature elements declared in the file are added to the current signature. In this case, the factors must be preceded by the keyword `theorems:`.
- ***add conjects*** opens a submenu of files, each one containing a sum of terms or a conjunction of formulas. They are added to the set of current conjectures. All conjectures are entered into the text field. Signature elements declared in the file are added to the current signature. In this case, the term or formula must be preceded by the keyword `conjects:`.

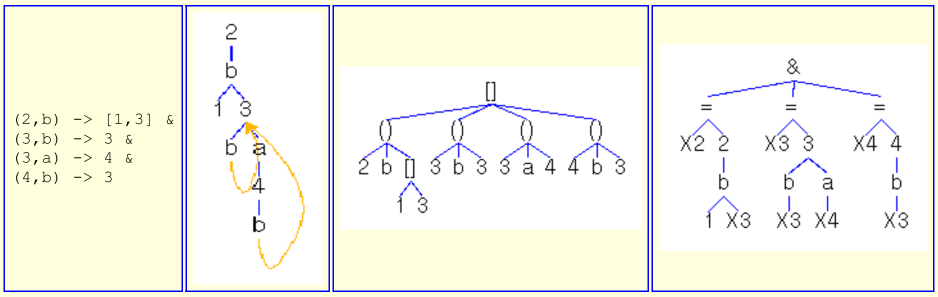
graph menu

- ***redraw (z)*** redraws the displayed tree. This removes junk from the canvas (see *Mouse and key events*).
- ***expand*** dereferences all pointers of the displayed tree or the selected subtrees, respectively. If the entry field contains a positive number n (default: n = 0), each circle in the trees is unfolded n times.
- ***expand leaves*** dereferences all pointers to leaves of the displayed tree or the selected subtrees, respectively.
- ***collapse -->*** and ***collapse <--*** identify all common subtrees of the displayed tree or the selected subtrees, respectively. If there is a number n in the entry field, cycles are unfolded n times. Otherwise cycles are not unfolded. *collapse* --> creates pointers to the right, *collapse* <-- produces pointers to the left.



```
(n < 6 & n `mod` 2 = 0 ==> n -> [n,n+1]) &
(n < 6 & n `mod` 2 =/= 0 ==> n -> n+1) &
6 -> [1,3,5,7..10]
```

Fig. 5A. *A transition system (TRANS0)*
as a conjunction of transitional axioms, a bipartite graph, a list of (state,successors)-pairs
and a conjunction of regular equations.



```
(2,b) -> [1,3] &
(3,b) -> 3 &
(3,a) -> 4 &
(4,b) -> 3
```

Fig. 5B. *A labelled transition system (TRANS1)*
as a conjunction of transitional axioms, a bipartite graph, a list of (state,label,successors)-triples
and a conjunction of regular equations.

- ***build list*** reverses the application of *build graph*, i.e. (1) a transition graph t has been selected or (2) a state term t of the form *Trans* or *TransL* is looked for in the current tree and t is compiled into an equivalent list of pairs consisting of a state and a list of states or into an equivalent list of triples consisting of a state s, a label and the list ofdirect successors of s. States and labels are arbitrary constants.

- ***build equations*** assumes that (1) the subtree t selected at last is a list of pairs consisting of a state s and the list of direct successors of s or a list of triples consisting of a state s, a label and the list of direct successors of s or (2) the current tree contains a state term t of the form *Trans* or *TransL*. *build equations* transforms t into an equivalent conjunction `X1=t1&...&Xn=tn` of **regular equations**, i.e. `X1,...,Xn` are variables and `t1,...,tn` are non-variable terms.
- ***build graph*** assumes that (1) the subtree t selected at last is a collection of pairs consisting of an integer state and a list of integers or a collection of triples consisting of an integer state, a constant label and a list of integers or a conjunction of regular equations or (2) the current tree contains a state term t of the form *Trans* or *TransL*. *build graph* transforms t into an equivalent transition graph. Edge labels are turned into node labels so that the graph is actually a bipartite one. The graph is constructed in a depth-first manner starting out from the first element of the list or conjunction. Hence only pairs, triples or regular equations, respectively, that are "reachable" from this element are taken into account!
- ***build Trans/TransL*** assumes that an occurrence of the leaf `Trans` or `TransL` has been selected in the current tree. Moreover, the current set of axioms is supposed to contain equations `states=t` (and `labels=u` if `TransL` was selected) where t and u simplify to lists of constants. If the button is pushed and the set of current axioms contains an equation `Trans=t` resp. `TransL=t`, then t is compiled into a transition function `f :: STATE -> [STATE]` resp. `f :: STATE -> String -> [STATE]`. If the set of current axioms does not contain such an equation, the axioms for -> that are applicable to elements of `states` resp. `states x labels` are compiled into f. f will map all integers that are not in `states` resp. `states x labels` to the empty list. `states`, `labels` and f become arguments of the state constructor `Trans` resp. `TransL` and the resulting state term replaces the selected leaf (see Simplifications).
- ***label graph with Atoms*** labels each (integer) state s of a transition graph by the atomic formulas assigned to s in the value of the state term *Atoms* (see Simplifications).
- ***greatest lower bound*** colors the root of the greatest lower bound of the selected subtrees in green.
- ***store graph*** translates a subterm `file(F,t)` into a widget graph (see the), saves its Haskell code to F and replaces `file(F,t)` by `file(F)` (see the *paint buttons* and the *pict type menu*).
- ***predecessors*** colors the predecessors of the roots of the selected subtrees in green.
- ***successors*** colors the successors of the roots of the selected subtrees in blue.
- ***variables*** colors the variables of the selected subtrees in blue.
- ***free variables*** colors the free variables of the selected subtrees in blue.
- ***label roots with entry*** labels the roots of the selected subtrees with the string in the entry field provided that the transformed subtrees are formulas if and only if the original ones are formulas. The changed labels are colored in blue.
- ***polarities*** colors the roots of all subtrees of the displayed tree. A root is colored in green if the subtree has positive polarity. Otherwise it is colored in red.
- ***positions*** replaces the nodes of the displayed tree by their tree positions. Each pointer position is labelled in red with the position of its target node.
- ***height numbers*** replaces the labels of the nodes of the displayed tree t by their tree levels (heights) within t.
- ***preorder numbers*** replaces the labels of the nodes of the displayed tree t by their preorder positions within t.
- ***heap numbers*** replaces the labels of the nodes of the displayed tree t by their heap order positions within t.
- ***coordinates*** shows the coordinates of the node labels of the displayed tree.

substitution menu

- ***add from text field*** adds to the current substitution the substitution that is given by the conjunction of equations in the text field.
- ***apply*** applies the current substitution to the selected subtrees of the displayed tree (or to the entire tree if no subtrees have been selected) and sets the current substitution to the empty one.
- ***rename*** assumes that the entry field contains a conjunction of equations x=y between variables. All occurrences of x in the selected subtrees are replaced by y.
- ***remove*** clears the current substitution.
- ***show*** enters the equations that represent the current substitution into the text field.
- ***show in text field of Solver1/2*** enters the equations that represent the current substitution of Solver2/1 into the text field of Solver1/2.
- ***show on canvas of Solver1/2*** displays the equations that represent the current substitution of Solver2/1 on the canvas of Solver1/2. The equations become the current trees of Solver1/2.
- ***show solutions*** writes the positions of the solved formulas resp. normal forms among the current trees into the label field.

parse buttons

parse up parses the string in the text field according to the grammar given below, initializes the list of current trees and the tree mode and displays the first element of the list on the canvas. Term graphs are implemented as objects of the instance `Term String` of the Haskell type

```
data Term a = V a | F a [Term a] | Actions (STATE -> String -> ActLR) | Atoms [String] (String -> [STATE]) |
              Dissect [(Int,Int,Int,Int)] | Finals (STATE -> Bool) | FinalsL (STATE -> String -> Bool) |
              Fix [[Int]] | MatchFailureC (Term a) | Matrix [STATE] (STATE -> STATE -> [(STATE,STATE)]) |
              MatrixL [STATE] (STATE -> STATE -> [([STATE],[STATE])]) | Trans [STATE] (STATE -> [STATE]) |
              TransL [STATE] [String] (STATE -> String -> [STATE])
type STATE = String
```

The constructor V encapsulates first-order variables, F encloses logical and non-logical function symbols and higher-order variables. The other constructors are called **state constructors** and will be explained later (see *Simplifications*).

parse up expects a term or formula built up of logical operators, signature symbols and further strings that are regarded as function symbols and displays its tree representation on the canvas. If the term resp. formula splits into numbered subexpressions, then only the ones whose numbers are listed in the entry field will be combined by <+> resp. & and displayed.

Given natural numbers `n1,..,nk`, strings of the form `pos n1 .. nk` are interpreted as pointers (see above). Only first-order variables and pointers are turned into objects built up with the constructor `V`. Subtrees whose root starts with the character @ are not displayed.

parse down computes the textual representation of the displayed tree resp. selected subtrees, connects them with the symbol in the entry field and writes the result into the text field provided that all selected subtrees are either terms or formulas.

narrow/rewrite buttons

By repeatedly pushing the unlabelled button right of the *match/unify* button narrowing resp. rewriting steps are performed on (selected) trees in a depth-first order until,

- (1) *if no subtrees have been selected:* at most one narrowing/rewriting step has been performed on the current trees or the execution time exceeds 300 seconds,
- (2) *if subtrees have been selected:* at most one narrowing/rewriting step has been performed on each selected subtree t or on the t enclosing atom if t is a term and the current tree is a formula,
- (3) *if no subtrees have been selected and the entry field contains a positive natural number n:* at most n narrowing/rewriting steps have been performed on the current trees or the execution time exceeds 300 seconds,
- (4) *if no subtrees have been selected and the entry field contains a positive natural number n followed by an 's':* narrowing/rewriting steps have been performed on the current trees and the execution time exceeds n seconds,
- (5) *if a subtree t has been selected and the entry field contains a positive natural number n:* at most n narrowing/rewriting steps have been performed on t or the execution time exceeds 300 seconds,
- (6) *if a subtree t has been selected and the entry field contains a positive natural number n followed by an 's':* narrowing/rewriting steps have been performed on t and the execution time exceeds n seconds.

In cases (5) and (6), sub*terms* are modified only if they *match* against *unconditional* equations. By pressing the ***match/unify*** button one changes between the (default) **match mode**, the **unify mode** and greedy versions of these modes that determine whether a potential redex is matched or unified against an axiom. The unify modes admit the instantiation of redex variables by non-variables, the match modes do not. Rewriting can only be performed in a match mode. Usually, all applicable axioms are applied in parallel and each solution of the guard of an applicable axiom leads to a reduct. In a greedy mode, only one - randomly selected - applicable axiom is applied, but, as in a non-greedy mode, different solutions of the guard of this axiom lead to several reducts.

Narrowing upon a predicate or copredicate p generalizes **linear resolution** to the simultaneous application of all axioms for p. Narrowing also generalizes rewriting from terms to formulas and admits the instantiation of redex variables by non-variable terms. These substitutions are supposed to build up solutions of the formula at the beginning of a narrowing sequence. Since each narrowing step applies the definition (axioms) of a predicate, copredicate or defined function, we have also used the term **unfolding** for narrowing steps [P00].

Applying all applicable (Horn) axioms for a predicate p or a defined function f simultaneously results in the replacement of the redex by the *disjunction* of their premises and equations representing the computed unifiers. Applying all applicable (co-Horn) axioms for a copredicate p simultaneously results in the replacement of the redex by the *conjunction* of their conclusions. Some equational or transitional axioms may be only partially unifiable with the redex. These are applied as well, but contribute to the reduct not with their premises resp. conclusions, but with equations representing the partial unifiers. This extension is called **needed narrowing** [AEH,P96] and ensures that the iteration of narrowing steps proceeding from supertrees to subtrees leads to all solutions of the current trees (see NEED).

A rewriting or narrowing step consists in the simultaneous application of all axioms for the outermost predicate, copredicate or defined function of the maximal subtree to which some axiom applies (if *rules* is empty) or some clause of *rules* applies (if *rules* is nonempty).

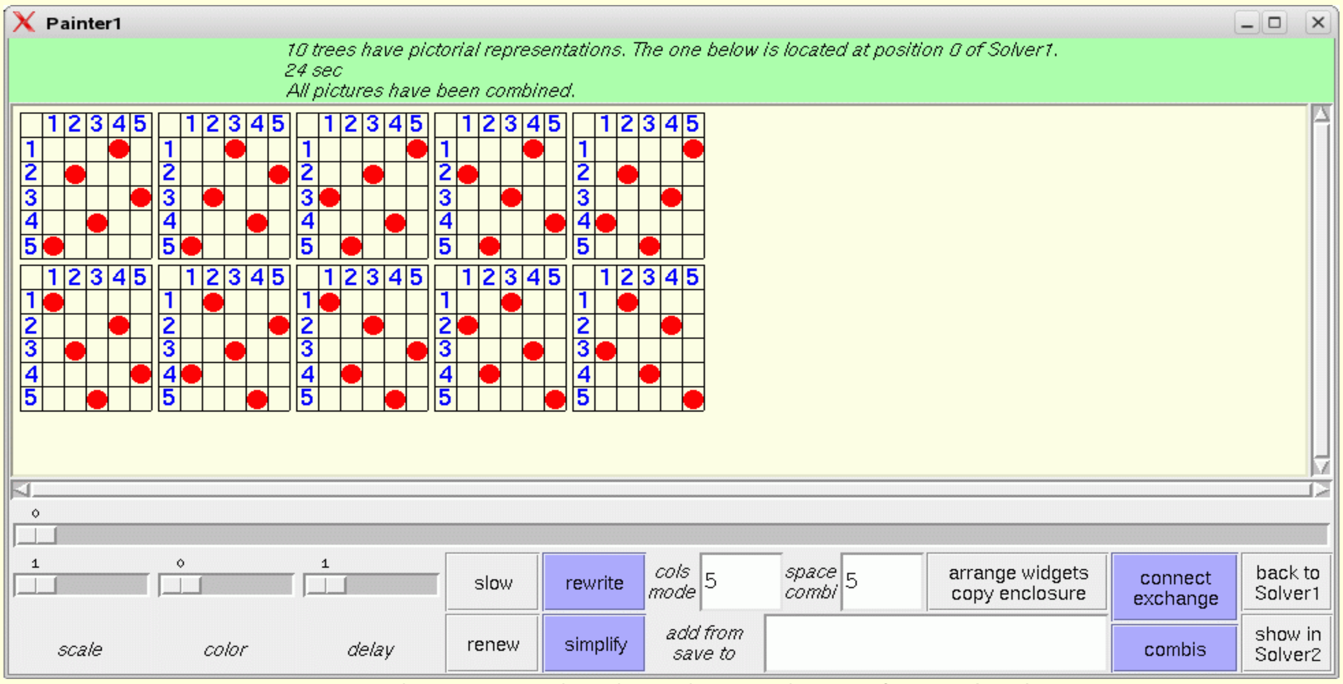
In cases (1), (3), (4), (5) and (6), reducts are simplified iff *simplifyBit* is set to `True` and simplified and refuted iff *removeBit* and *simplifyBit* are set to `True` (see *Solver state variables*).

simplify button

simplify performs simplification steps on (selected) trees from top to bottom and, on each level, from left to right, until,

- (1) *if no subtrees have been selected:* at most 100 simplification steps have been performed,
- (2) *if subtrees have been selected:* at most one simplification step has been performed on each selected subtree,
- (3) *if no subtrees have been selected and the entry field contains a positive natural number n:* all current trees are simplified or the number of successive simplification steps exceeds n,
- (4) *if a subtree t has been selected and the entry field contains a positive natural number n:* t is simplified or the number of successive rewriting/narrowing steps on t exceeds n.

paint buttons


Painter1

10 trees have pictorial representations. The one below is located at position 0 of Solver1.
24 sec
All pictures have been combined.

Fig. 6. *The painter window shows the ten solutions of queens(5,ps) obtained from applying axioms of QUEENS.*

Pictorial term representations consist of **widgets**. A list of widgets is called a **picture**. A picture becomes a **widget graph** if some of its widgets are connected by directed arcs. Widgets comprise circles, paths, polygons, text entries, node-labelled trees, and sequences of **turtle actions** that admit the hierarchical construction of pictures insofar as the drawing of a picture (without arcs) is also a turtle action.

The actual widget interpreter is selected from the *pict type menu*. Some built-in axiom files (see *Examples*) and enumerators are automatically associated with a widget interpreter.

paint fast/slow opens a *Painter* window with the background color entered in the solver's entry field (see the *Grammar*; white is the default color) that consists of the following widgets:

- A label field for displaying messages.
- A scrollable canvas.
- A slider for selecting the graph to be displayed on the canvas.
- The **scale** slider sets the scaling factor of the displayed widgets. The font and size of text widgets must be the set in the window of the solver from which the painter was called. It is not changed by moving the scale slider! If *cols/mode* contains the string `s`, the painter enters the **space mode**: subsequent moves of the *scale slider* do not change the scaling of the current graphs. Instead, movements of the *scale slider* enlarge resp. reduce the space between adjacent widgets. The space mode is left when `s` is removed from the *cols/mode* field before the scale slider is moved.
- The **color** slider modifies the colors of the displayed widgets. Moving the slider by n units to the right/left changes color c to color c+n/c-n with respect to a circular list of 306 equidistant colors (see *Widget interpreters*).
- The **delay** slider selects the time interval (from 1 up to 300 milliseconds) between the paintings of two successive widgets of the displayed picture.
- The **fast/slow** button changes between immediate and delayed display. In the first case, the widgets of the displayed picture are painted concurrently. In the second case, a widget is drawn n milliseconds after its predecessor has been painted where n is set with the *delay* slider. A widget prefixed by the constructor *fast* is always painted fast (see *Widget interpreters*).
- **renew** calls *paint* from the painter window.
- **narrow/rewrite** and **simplify** induce the synonymous actions in the associated solver and display their pictorially representable results on the canvas of the painter.
- The **cols/mode** entry field takes a parameter for *arrange widgets* or *connect* actions (see below).
- The **space/combi#** entry field takes a number that is interpreted as
  - the space between adjacent rows or columns of the matrix of widgets that is displayed if the *arrange widgets* button is pushed,
  - the number of times the permutation successor function `suc` is applied if *cols/mode* contains the string `p` (see below),
  - the number of the representation of the current graph to be displayed (see below) if the *combis* button is pushed.
- If the **arrange widgets/copy enclosure** button is pushed and no subgraph is enclosed, then the painter reads the entries of the *cols/mode* and *space/combi#* fields and performs one of the following actions. Let d and e be the (real) number in the *space/combi#* field and *spread=(hor,ver)* (see *Solver state variables*). If the *space/combi#* field does not contain a real number, let d=0 and e=1.
  - If *cols/mode* contains the string `s`, the painter enters the **space mode**: subsequent moves of the *scale slider* do not change the scaling of the current graphs. Instead, movements of the *scale slider* enlarge resp. reduce the space between adjacent widgets. The space mode is left when `s` is removed from the *cols/mode* field.
  - If *cols/mode* contains the string `q`, the painter enters **arrange mode q**: the widgets of the current graphs are arranged in a square matrix with a space of d units between adjacent widgets.
  - If *cols/mode* contains a positive natural number n, the painter enters **arrange mode n**: the widgets of the current graphs are arranged in a matrix with n columns and a space of d units between adjacent widgets.
  - If *cols/mode* contains the string `t`, the painter enters **arrange mode t**: the current graphs are turned into a forest of rooted graphs with a horizontal space of 3e*hor units between adjacent siblings and a vertical space of d*ver units between adjacent tree levels.
  - If *cols/mode* contains the string `b`, the painter enters **arrange mode b**: the current graphs are turned into a forest of rooted graphs with a horizontal space of 3e*hor units between adjacent siblings and a vertical space of d*ver units between adjacent tree levels. Moreover, each non-tree edge of the subgraph is displayed as a B-splined arc with a red control point for interactive reshaping.
  - If *cols/mode* contains the string `c`, the painter enters **arrange mode c**: the current graphs are turned into a forest of rooted graphs with a horizontal space of 3e*hor units between the roots of adjacent subgraphs at the same tree level and a vertical space of d*ver units between adjacent tree levels. The nodes of the graphs are centered around a vertical axis.
  - If *cols/mode* contains the string `p`, the nodes of the current graphs are reordered by applying the permutation successor function `suc` one or more times (see *Built-in signature*). The modified graphs are displayed according to m. If the painter is in arrange mode t, b or c, the in- or outgoing arcs of each widget w become in- resp. outgoing arcs of w, which takes its place after the widget list has been permuted.Otherwise each widget takes its adjacent arcs to its new position.
  - If *cols/mode* contains the empty string, the current arrange mode is left.
  - If the button is pushed and a subgraph G is enclosed, a copy of G is placed to the right of G.
- The **add from/save to** entry field takes the name of a file
  - containing a graph in Haskell code that is added to the displayed graph if the cursor is in the field and the *Up* key is pushed,
  - the displayed graph is saved to in Haskell code or eps format (if the file has the suffix *.eps*) if the cursor is in the field and the *Down* key is pushed (see *Widget interpreters*).
    Files with Haskell code are looked up in resp. saved to the *Examples* directory, eps files are saved to the *Pics* directory (see *Main commands*).
- The **connect/enclose** button switches to or from a state in which one of the following actions is performed.
  1. If the left mouse button is pushed and moved from widget w to widget w' and the *cols/mode* field does not contain the string `e`, a directed arc is drawn from w1 to w2.
  2. If the right mouse button is pushed and moved from widget w to widget w' and the *cols/mode* field does not contain the string `e`, a B-splined directed arc with a red control point is drawn from w1 to w2.
  3. If the left mouse button is pushed and moved from widget w to widget w' and the *cols/mode* field contains the string `e`, w1 and w2 are exchanged. If the painter is in arrange mode t, b or c, the in- or outgoing arcs of wi become in- resp. outgoing arcs of wj where (i,j) ∈{(1,2),(2,1)}. Otherwise w1 and w2 take their adjacent arcs to their new positions.
  4. If the middle mouse button is pushed and moved from northwest to southeast, a rectangle is drawn whose upper-left and lower-right corners agree with the cursor positions when the mouse button was pressed resp. released.

- The **combis** button browses through 15 representations of the current graph:
  1. The red **control points** of B-splined arcs **are hidden**. The same holds true in the following representations.
  2. **Bipartite coloring**: Nodes at even levels of the widget graph are colored with some color c, nodes at odd levels are colored with the complement color of c.
  3. **Rainbow coloring**: The root of the widget graph is colored with some color c. Nodes at level n+1 are colored with the successor of the color of nodes at level n with respect to a circular list of n equidistant colors (see *Widget interpreters*) where n is the height of the current tree the graph is constructed from.
  4. The displayed graph is extended by blue arcs that form the **convex hull** of the graph's set of nodes.

5. In addition to 4, non-text hull nodes are numbered counter-clockwise.
6. The **anchor** of each displayed widget is shown as a white or grey point depending on whether the anchor is or is not part of another widget.
7. The displayed graph is extended by a black dot at each **crosspoint** between two widgets.
8. The displayed graph is extended by colored lines enclosing the polygons that form **intersections** of overlapping widgets. The color of a line is the dark version of the color of the widget the line belongs to.
9. The displayed graph is extended by colored lines enclosing the polygons that form **unions** of overlapping widgets. The color of a line is the dark version of the color of the widget the line belongs to.
10. The displayed graph is extended by white polygons that form **intersections** of overlapping widgets.
11. Same as 10 except that an intersection polygon p is colored grey if the widget w p belongs to is white. Otherwise p is colored with a light version of the color of w.
12. Same as 10 except that the intersection polygons are colored in a way that leads to the impression that they are **weaved** into each other.
13. The displayed graph is extended by white polygons that form **unions** of overlapping widgets.
14. Same as 13 except that the, say, n **holes** of the union polygons are **colored** differently. For each two adjacent holes h and h', the color of h is the successor of the color of h' with respect to a circular list of n equidistant colors (see *Widget interpreters*).
15. Combination of 10 and 13.

If the *combi#* field contains a number n between 0 and 12, the n-th representation in the above list will be displayed. To ensure that all parts of the representation are visible, press the *slow* button!
If the *partition* interpreter has been selected (see the *pict type menu*), the **combis** button browses through 6 colorings the rectangles representing the current tree:

1. The color of a rectangle is determined by the tree level (height) of the leaf that the rectangle represents.
2. Same as 1 except that two adjacent leaves are represented by rectangles whose colors are most distant from each other.
3. The color of a rectangle is determined by the preorder position of the leaf that the rectangle represents.
4. Same as 3 except that two adjacent leaves are represented by rectangles whose colors are most distant from each other.
5. The color of a rectangle is determined by the heap order position of the leaf that the rectangle represents.
6. Same as 5 except that two adjacent leaves are represented by rectangles whose colors are most distant from each other.

- **back to Solver1/2** closes the painter window and opens the Solver1/2 window.
- **show in Solver2/1** button constructs a rooted graph from the displayed widget graph and shows it in the Solver2/1 window. The nodes of the rooted graph are labelled with the positions of the widget in the list of widgets repesenting the widget graph.
- **undo** revokes the immediately preceding action on the displayed graph. Note that *undo* cancels only the last step and there is no *redo* button for revoking *undo*.
- **stop/go** interrupts/resumes drawing.

The commands for creating and editing widget graphs are summarized in Figures 20 and 21 (see *Widget interpreters*).

If subtrees have been selected, *paint* combines the pictorial representations of all representable selected subtrees and displays the resulting picture on the canvas. Using the middle mouse button, the individual widgets, which are usually displayed on top of each other, can be pulled from each other horizontally or vertically.

If no subtrees have been selected, then for each element t of the list of current trees, *paint* combines the pictorial representations of all maximal representable subtrees of t. The resulting picture that corresponds to the tree displayed on the solver canvas is drawn on the painter canvas, while the pictures derived from other elements of the list of current trees are assigned to other positions in the **list of current pictures**. One may browse among the pictures by moving the graph selecting slider (see above). Expander2 provides several widget interpreters and combinators thereof. The actual one depends on the current axioms, but can also be set by selecting from the *pict type* menu. For instance, Fig. 7 shows solutions of the formula

```
Any pa: (loop((0,0),path[],pa) & turt(pa:place(circ(2,red),[(2,6),(6,2)])) = z)
```

in `ss` obtained by applying axioms of *robot*, were generated by the interpreter *polygon solution* that looks for solved formulas and applies the interpreter *polygon* to the solving terms in these formulas. A solved formula looks as follows:

```
Any Z1:x1=t1 &...& Any Zk:xk=tk & All Z(k+1):x(k+1)=/=t(k+1) &...& All Zn:xn=/=tn.
```

x1,...,xn are different free variables, t1,...,tn are normal forms and the transitive closure of {(i,j)|ti contains xj} is acyclic.

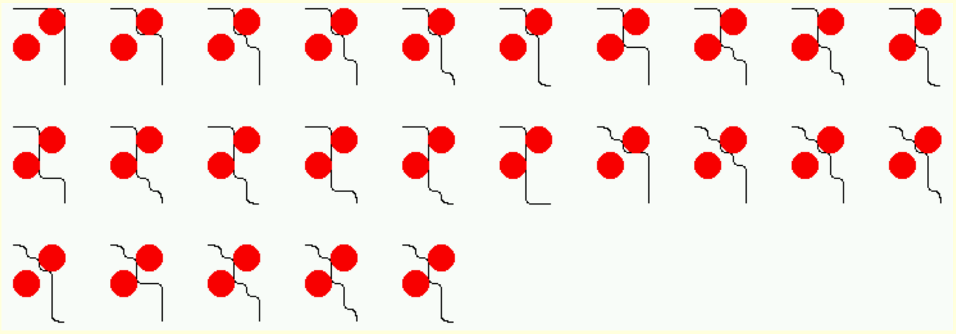


Fig. 7. *Pictorial representations of solutions obtained by applying axioms of robot*

Further buttons

- ***split/join*** decomposes a conjunction, disjunction or sum into its factors, summands and terms, respectively, provided that there is only current tree, i.e. the state variables *oneTree* and *treeMode* have the value *True* and *tree*, respectively. If *oneTree=False* and the value of *treeMode* is *summand*, *factor*, or *sum*, then the current trees are combined into a single disjunction, conjunction or sum, respectively.

- ***hide/show*** hides all selected subtrees if the state variable *hideState* is set to *True*. Otherwise the hidden parts of the selected subtrees are shown. If *hide/show* is pushed while no subtrees are selected, the Boolean value of *hideState* changes.

  Trees with hidden subtrees can also be simplified, narrowed or rewritten. This saves time because the reducts of hidden subtrees are not displayed and is particularly useful when tree transformations are called from the painter for the purpose of interpreting their results immediately as pictures.

- ***+1/-1*** increases/decreases the natural number in the entry field. If the entry field does not contain a natural number, then *+1* writes *0* into it, while *-1* clears it.
- ***set*** selects the subtrees whose positions in heap order agree with the numbers in the entry field.
- ***remove text*** clears the text field.
- ***remove entry*** clears the entry field.
- ***clear subtrees*** deselects all selected subtrees.
- ***collapse levelwise*** identifies the common subtrees of the current tree level by level.
- ***apply to variable:*** opens a menu of all variables in the domain of the current substitution f. If the button for a variable x is pressed, all occurrences of x in the subtree selected at last are replaced by f(x). If they are bound by an existential/universal quantifier and the respective quantified subformula t has positive/negative polarity, then all occurrences of x in t are replaced by f(x) and x is removed from the quantifier.
- ***<---/--->*** proceed one step backward/forward in the current proof and display the corresponding list of current trees. As soon as a rule is applied to the list, its previous successors are removed from the proof.
- ***derive/stop*** switches between
  - the (default) label "derive" indicating that *removeBit* and *simplifyBit* are set to `False`,
  - the label "derive & simplify" indicating that *removeBit* is set to `False`, *simplifyBit* is set to `True` and thus automatic simplification steps are performed after each *apply clause*, *coinduction*, *fixpoint induction*, *instantiate*, *narrow/rewrite* or *replace by other sides* step (see *Simplifications*),
  - the label "derive & simplify & refute" indicating that *removeBit* and *simplifyBit* are set to `True` and thus the additional removal of non-narrowable/non-rewritable subtrees during *narrow/rewrite* steps,
  - the label "stop" indicating that a proof term is going to be evaluated. If the button is pushed in this state, the not-yet-evaluated part of the proof term is removed and the current proof may be continued differently.
- ***increase/decrease current*** proceed to the next/previous element of the list of current trees if the up resp. down key is pushed after the label field has been activated. If a painter window is in the foregound, then the next/previous picture is displayed.
- ***quit*** quits Solver1/2.

Grammar

according to which *parse up* translates a string in the text field into a term or formula. Bold symbols are terminal.

implication ---> disjunct **==>** disjunct | disjunct **<==>** disjunct | disjunct **===>** disjunct | disjunct **<===** disjunct

disjunct ---> conjunct | conjunct **|** disjunct

conjunct ---> enclosedFactor | enclosedFactor **&** conjunct

enclosedFactor ---> **(**implication**)** | factor

factor ---> **True** | **False** | **Not** enclosedFactor | **Any** vars : enclosedFactor |

All vars : enclosedFactor | infixAtom | prefixAtom | singleTerm^singleTerm moreBag

vars ---> var | var vars

var ---> noBlanks *noBlanks must derive to a first- or higher-order variable.*

infixAtom ---> term infixToken term *infixToken must derive to* **=**, **=/=** *or a predicate or copredicate.*

prefixAtom ---> noBlanks | *noBlanks must derive to a first-order variable.*

noBlanks atomrest | *noBlanks must* derive to **=**, **=/=**, *a predicate or a copredicate.*

(infixRelTerm**)** someTerms | **(**infixRelTerm**)**

atomrest ---> **(**relTerms**)** someTerms | enclosedTerms manyTerms

someTerms ---> **(**relTerms**)** someTerms | enclosedTerms someTerms | enclosedTerms

relTerms ---> relTerm | relTerm **,** relTerms

relTerm ---> prefixRelTerm | infixRelTerm | term

prefixRelTerm ---> preRelChars | preRelChars **(**relTerms**)** |

preRelChars must derive to **=**, **=/=**, *a predicate or a copredicate.*

infixRelTerm ---> **(**enclosedRelTerm infixToken enclosedRelTerm**)**

infixToken must derive to a constructor or a defined function.

enclosedRelTerm ---> **(**relTerm**)** | prefixRelTerm

enclosedTerms ---> **(**terms**)** | list | set | **(**infixFun**)**

terms ---> term | term **,** terms | term **..** terms

term ---> bagTerm | bagTerm **<+>** term

bagTerm ---> singleTerm moreInfix | singleTerm moreInfix **^** bagTerm

moreInfix ---> **+** singleTerm moreInfix | **-** singleTerm moreInfix | infixFunR bagTerm | *empty*

+ *and* **-** *are left-associative.*

infixFun ---> infixToken *infixToken must derive to a constructor or a defined function.*

infixFunR ---> infixFun

infixFun must derive to a right-associative function, but not to **+**, **-**, **^** or **<+>**.

```
singleTerm ---> list | set | boolTerm | int | int curryrest | double | string | fovar | #dddddd |

RGB int int int | pos treepos | -singleTerm | () | (term) curryrest |

enclosedTerms | noDelims | noDelims curryrest

noDelims must not derive to =, =/=, a logical symbol, a predicate (except for _),

 a copredicate (except for _) or a first-order variable.

curryrest ---> enclosedTerms curryrest | empty

boolTerm ---> bool(implication) | cond(implication,terms) | lin(conjunct)

list ---> [] | [terms]

set ---> {} | {terms}

int ---> any constant of Haskell type Int

double ---> any constant of Haskell type Double

string ---> any string

fovar ---> noBlanks noBlanks must derive to a first-order variable.

treepos ---> any finite list of natural numbers separated by blanks

infixChars ---> any string that consists of characters among . ; : + - * < = ~ > / \ ^ #

infixWord ---> `any string that does not contain back quotes`

infixToken ---> infixChars | infixWord | ~k | ~/~k for all natural numbers k

noDelims ---> any string that does not contain a character among

 ( ) [ ] { } , ` | & . ; : + - * < = ~ > / ^ # \t \n

noBlanks ---> any string that neither contains a blank or a character among

 ( ) [ ] { } , ` | & . ; : + - * < = ~ > / ^ # \t \n

 relChars ---> any string that neither contains a blank nor a character among ( ) , \t \n

d ---> 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D | E | F
```

`noDelims` need not derive to a symbol of the current signature. Any string derived from `noDelims` is turned into a node with the `Term` constructor `F` (see the *parse buttons*). Moreover, `noDelims` may derive to a string with blanks. This permits the use of symbols consisting of several words separated by blanks.

Integers, reals and (quoted) strings are automatically interpreted as (not always nullary!) constructors. This admits, for instance, the use of natural numbers in the tree representations of nested partitions (see *Dissections and partitions*).

An important technical reason for declaring a function symbol as a defined function is the fact that the outermost non-equational symbol of each axiom must be a predicate, a copredicate or a defined function.

Newline characters followed by a dot must be avoided because this because such a string is interpreted in a particular way. When a line with more than 120 characters is entered into the text field, it is split into several lines each of which starts with a dot. This ensures that decomposed lines are recognized as single ones when the contents of the text field is parsed.

Line suffixes starting with `--` are regarded as comments.

If the prefix x of a string x_y is parsed into a color c according to the following grammar, then x_y will be displayed on the canvas of a solver as a c-colored y.

The **color grammar**:

```
      color ---> light color | dark color | RGB int int int | #dddddd
      color ---> black | grey | white | red | magenta
      color ---> blue | cyan | green | yellow | orange
      int   ---> any constant of Haskell type Int
      d     ---> 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D | E | F
```

Axioms and theorems

Axioms and theorems to be applied with *apply clause* must have a *stretchable* premise or conclusion or must be
Horn clauses:

```
      (1) {guard ==>} (f(t1,..,tn)) = u {<=== prem})
      (2) {guard ==>} (p(t1,..,tn) {<=== prem})
      (3) t = u {<=== prem}
      (4) q(t1,..,tn) {<=== prem}
```

co-Horn clauses:

```
      (5) {guard ==>} (q(t1,..,tn) ===> conc)
      (6) t = u ===> conc
      (7) p(t1,..,tn) ===> conc
```

distributed Horn clauses:

```
      (8) at1 | ... | atn {<=== prem}
      (9) at1 & ... & atn {<=== prem}
```

distributed co-Horn clauses:

```
      (10) at1 | ... | atn ===> conc
      (11) at1 & ... & atn ===> conc
```

`f`, `p` and `q` denote a defined function, a predicate and a copredicate, respectively, of the current signature. If the current trees are terms, then the reducts must be terms and thus only premise-free clauses of the form (1) can be applied.

A clause with a guard is applied only if the guard is solvable. The solution becomes part of the unifier that is generated when the clause is applied. For instance, the axiom

```
 split(s) = (s1,s2) ==> sort(x:(y:s)) = merge(sort(x:s1),sort(y:s2)),
```

for sort (see LISTEVAL) is guarded, while the logically equivalent axiom

```
sort(x:(y:s)) = merge(sort(x:s1),sort(y:s2)) <=== split(s) = (s1,s2)
```

(see LIST) is unguarded. On the one hand, guarded axioms are needed for evaluating ground terms efficiently. On the other hand, axioms and theorems used as lemmas in step-by-step derivations (see below) must be unguarded. Otherwise the search for a solution of the guard may block the derivation process.

Narrowing is used for solving guards. In order to ensure termination, at most 100 narrowing steps, each of which followed by at most 100 simplification steps, are performed for solving a guard.

Axioms are of type (1), (2) or (6). The *step functions* (or *consequence operators*) induced by axioms must be monotone [P00,P05]. Usually, `f`, `p` resp. `q` agree with the root of the redex to which a clause is applied.

For applying a non-distributed clause, select a term/atom `at'` with positive/negative polarity in the displayed tree such that the leading term/atom `at` is unifiable with `at'`. `at'` is replaced by the corresponding instance of `prem`/`conc`.

For applying a distributed clause cl, select n atoms `at1',...,atn'` in a disjunction/conjunction F with positive/negative polarity of the displayed tree such that for all 1 <= i <= n, `ati` is unifiable with `ati'`. The summands/factors of F where `at1',...,atn'` are selected from must not contain universal/existential quantifiers or negation or implication symbols. `at1',...,atn'` are replaced by the corresponding instance of `prem`/`conc`. The resulting summands/factors are combined conjunctively if cl is a Horn clause and disjunctively if cl is a co-Horn clause (see [FMS,P02]).

When a formula F with a stretchable premise/conclusion is selected for application, then the premise/conclusion of F is streched and thus F is turned into a co-Horn/Horn clause (see *fixpoint induction* for premise stretching and *coinduction* for conclusion stretching), which is then applied as in case (3), (4), (6) or (7), respectively.

Derivations

A proof with Expander2 is a sequence of successive values of the state variable *trees*. It is documented and stored in the state variable *proof*. The values of *proof* and *current trees* are initialized whenever *parse up* parses the contents of the text field and displays the resulting tree t on the canvas. Then *trees* is set to [t], *tree* is set to t, and *proof* is set to the initial values of its components.

A proof step is correct if the transformed disjunction (or conjunction or sum) of the current trees implies (or is equivalent to) the original one. In the case of possible incorrectness Expander2 delivers the warning

```
CAUTION: This step may be semantically incorrect.
```

Such steps are not stored in the current proof term.

If the current trees are formulas, a proof ending up with `True` or `False` yields a proof resp. refutation of the conjunction/disjunction of the initial trees. Other final results are given by *solved formulas* that represent solutions of the original conjecture in their free variables.

Axioms of the form `{guard ==>} t -> u {<=== prem}` can only be applied to left-hand sides of *transitional atoms* `t' -> v`. If t and t' are unifiable, then `t' -> v` is replaced by the corresponding instance of the formula `u = v {& prem}` provided that the unifier satisfies `guard` (see *Built-in signature*).

If the current trees are terms, the application of an axiom yields a *rewriting step* from a term to a term. Hence only axioms of the form `{guard ==>} t = u` or `{guard ==>} t -> u` may be applied: the term to be rewritten is matched against t replaced by the corresponding instance of u provided that the corresponding instance of `guard` is solvable (by narrowing). If several axioms are applicable to the same term, they are applied in parallel and the reducts are combined by `<+>` to a *sum term*. For semantical reasons, the latter should only happen if an axiom of the form `{guard ==>} t -> u` is applied.

A higher-order variable F may be substituted when terms or formulas are matched or unified, but only if the symbol g substituted for F is admissable for F (see *Built-in signature*) and either the outdegrees of the nodes m and n labelled with F resp. g are equal or the outdegree of m is 0 and the entire subtree with root n is substituted for m. The correctness of a proof step depends on the **polarity** of the redex with respect to its position within the displayed tree. The polarity is *positive* if the number of preceding negation symbols or premise positions is even. Otherwise the polarity is *negative*. Fixpoint induction, coinduction, summand removal, summand unification, applications of Horn clauses, instantiations of existential variables and term replacements (see *replace by other sides*) are correct if the redex has positive polarity because here the reduct implies the redex. Atom composition, factor removal, factor unification, applications of co-Horn clauses and instantiations of universal variables are sound if the redex has negative polarity because here the redex implies the reduct. Simplification, rewriting, narrowing in a unify mode (see the *narrow/rewrite buttons*), splitting, flattening and stretching may be applied to any possible redex within the displayed tree because here the redex and the reduct are equivalent. Narrowing in a match mode may also be applied to any possible redex. However, if a narrowing redex unifies, but does not match with some axiom, narrowing in a match mode is stopped with a corresponding message. All these restrictions ensure that the resulting formula implies the original one.

The theorem-proving features of Expander2 do not aim at fully automatic proofs. Expander2 favors natural deduction in contrast to many other provers that submit a conjecture to Skolemization and other extensive normalizations before the proof can start. This restricts the readability and thus the controllability of derivation processes significantly, especially when induction or coinduction steps are involved that are at the heart of any non-trivial program verification. Fortunately, the axioms, the theorems and, to some extent, the conjectures we are faced with in program verification already come as Horn or co-Horn clauses and thus can indeed be handled by Expander2 in their original form.

It also complies with a natural proof process that Expander2 avoids negation symbols. The simplifier drives them innermost until they directly precede (co)predicates and can be removed completely by transforming the (co)predicates into their complements (see *negate axioms for symbol*). Negation-free axioms induce monotone *consequence operators*. Hence predicates and copredicates have least resp. greatest interpretations in the initial model of the underlying specification [P00,P05].

Variables

Variables of a clause that are introduced into the current tree when the clause is applied to the tree are renamed by increasing the number suffixes of the variables. Variables that the tree shares with the applied clause are renamed in the same way. Since variable renaming affects the *state variable* *varCounter*, it is not performed during simplification.

Variables of a Horn or co-Horn clause are turned into existential resp. universal variables. The scope of these variables is the respective reduct.

If a free variable x of a redex is instantiated by a term t during a rewriting or narrowing step, then the equation x=t is added to the reduct.

Simplifications

Narrowing removes predicates, copredicates and non-constructor functions from the current trees. The simplifier does the same with logical operators, constructors and *built-in symbols*. Simplifications realize the highest degree of automation and the lowest level of interaction (see *Overview*). Pushing the *simplify* button admits step-by-step simplification of the current trees. The rules applied by the simplifier employ not only logical equivalences, but also the semantics of constructors, equality or inequality predicates and other *built-in symbols*. For instance, an implication `prem==>conc` is reduced to `True` if `prem` **subsumes** `conc`, a disjunction is reduced to its minimal summands, a conjunction to its maximal factors. Here are some examples:

```
Any x y z:(x=f(y) & Q(z)) ==> Any x' y' z':(Q(z') & f(y')=x')
```

reduces to `True`.

```
Any x:Q(x) & Q(suc(y)) & All x:R(x) & R(y+z) & Any x:x=suc(y) & suc(y)=y+z
```

reduces to `Q(suc(y)) & All x:R(x) & suc(y)=y+z`.

```
 Any x: (x = f(h(y,z),z) & P(x,y))
& All x: (x =/= f(h(y,z),z) | P(x,y))
& All x: (x = f(h(y,z),z) & P(x,y) ==> Q(x))
& All x: (P(x,y) ==> x =/= f(h(y,z),z) | Q(x)).
```

reduces to `P(f(h(y,z),z),y) & Q(f(h(y,z),z))`.

```
P(x,y) & Q(z) & (P(x,y) ==> R(x,y,z))
```

reduces to `P(x,y) & Q(z) & R(x,y,z)`.

```
P(x,y) ==> (Q(y) ==> R(x,y,z)) | P(y,z)
```

reduces to `P(x,y) & Q(y) ==> R(x,y,z) | P(y,z)`.

```
2 `in` [1,2,3]
```

reduces to `True`.

```
y `in` [1,x,4]
```

reduces to `y=1 | y=x | y=4`. The example shows that an atom of the form `` t`in`ts `` where t and ts are constructor terms is simplified to a solved formula and can thus be used in the guard of an axiom.

```
[2,3]++[5`mod`2,1] <+> 78 <+>{}^{9,5,5}^{9,9,5}
```

reduces to `[2,3,1,1] <+> 78 <+> {}^{5,9}^{5,9}`.

```
[1,2,3]-2
```

reduces to `[1,3]`.

```
zipAny(=)[1,x,3,4][5,2,y,6]
```

reduces to `x=2 | 3=y`.

```
zipAny(=)[1,x,3,4][1,2,y,6]
```

reduces to `True`.

```
zipAll(=)[1,x,3,4][1,2,y,4]
```

reduces to `x=2 & 3=y`.

All rules applied by the simplifier and dealing with logical operators and equality or inequality predicates are listed in [Prover], section 5. This section also contains a defintion of subsumption (see above) that captures almost all implications whose validity follows from their syntactic structure.

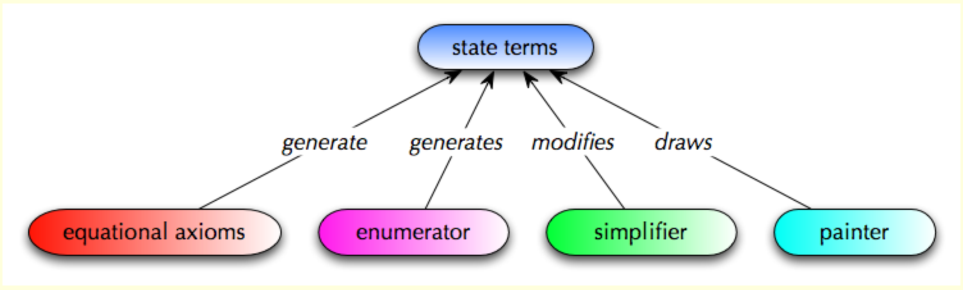


Fig. 8. *The environment of state terms*

Some commands induce the creation of **state terms**. A state term consists of a *state constructor* and attributes of various types and may occur as part of any other term or formula (see the *parse buttons*). If the simplifier encounters a state term t in the current tree, it calls a built-in Haskell function f that operates on the attributes of t and assigns new values to the attributes of t. The entire current tree is modified accordingly. The attributes of a state term do not appear on the canvas or in the text field of a solver. However, the painter may be able to translate them into pictures.

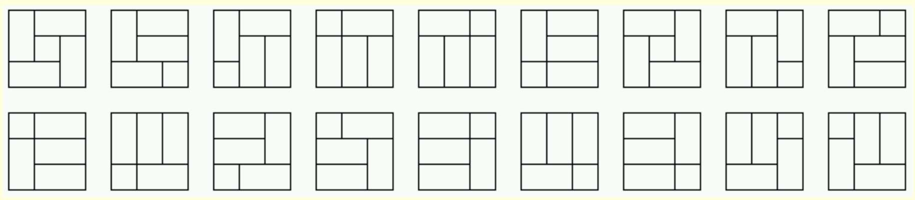


Fig. 9. *All dissections of a 3x3-rectangle that satisfy area(2):*
each dissection consists of ceiling(9/2)=5 subrectangles.

The following Haskell programs executed (stepwise) by the simplifier use and modify state terms:

- **auto** computes a nondeterministic automaton from a regular expression;
- **bisim** computes bisimilar states of a labelled transition system by table filling [HMU];
- **gauss** solves linear equations by applying the Gaussian algorithm;
- **nerode** computes behaviorally equivalent states of a deterministic Moore or Mealy automaton by table filling [HMU];
- **parse** runs an LR(1) parser with respect to given transition and action tables;
- **permute** permutes the list of Boolean variables of a DNF or OBDD;
- **postflow** verifies an iterative program by the backward propagation of a postcondition through its flowgraph;
- **sat** computes the set of states satisfying a given CTL formula;
- **stateflow** computes the set of states satisfying a given μ-calculus formula F by the backward propagation of state sets through a flowgraph that represents F (the validity of F in a given state can be proved by applying induction, coinduction and narrowing with respect to the axioms of CTL or LTL);
- **subsflow** interprets an iterative program by the forward propagation of sets of substitutions through its flowgraph.

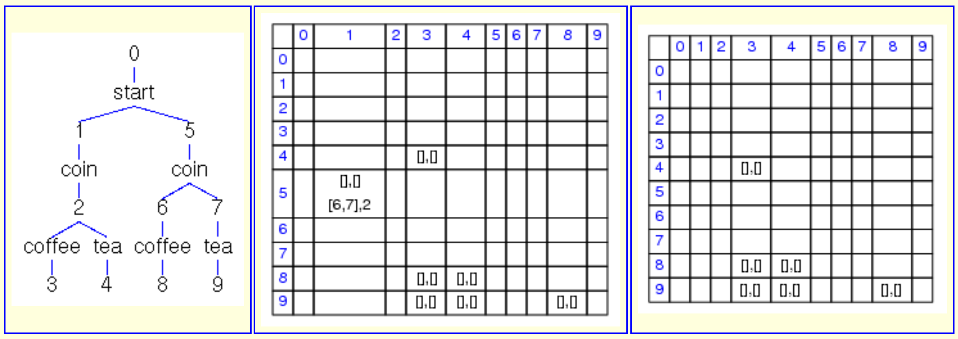
The programs receive their input by initializing the associated state terms. The initialization consists in rewriting synomymous constants upon particular equational axioms or calling build Trans/TransL:

- The constant `Actions` is rewritten to the state term `Actions f` by applying an axiom `Actions=t` where t simplifies to a collection of triples (s,x,a) consisting of a state (constant) s, a terminal symbol (constant) x and an action (constant represented as a string with blanks) a. x may be the empty word (denoted by the string *end* or equal to the string *other*. a must be equal to *shift* or *error* or denote a grammar rule r, given by a sequence of constants separated by blanks: the first, second and remaining constants are regarded as the name, the left-hand side and the right-hand side of r, respectively. A triple (s,*other*,a) of u stands for the list of triples (s,x1,a),...,(s,xn,a) where {x1,...,xn} is the set of terminal symbols x =/= *other* for which no triple (s,x,b) occurs in u. t is complied into `f :: STATE -> String -> ActLR`.
- The constant `Atoms` is rewritten to the state term `Atoms atoms f` by applying an axiom `Atoms=t` where t simplifies to a collection of pairs consisting of a state (constant) and a list of state predicates (constants). t is complied into `atoms :: [String]` and `f :: String -> STATE`.
- The constant `Finals` is rewritten to the state term `Finals f` by applying an axiom `Finals=t` where t simplifies to a list of states (constants). t is complied into `f :: STATE -> Bool`.
- The constant `FinalsL` is rewritten to the state term `FinalsL f` by applying an axiom `FinalsL=t` where t simplifies to a collection of pairs consisting of a state (constant) and a label (constant). t is complied into `f :: STATE -> String -> Bool`.
- The constant `Trans` is rewritten to the state term `Trans states f` by applying an axiom `Trans=t` where t simplifies to a collection of pairs consisting of a state (constant) and the list of its direct successors (constants; see the matrix interpreter in *Widget interpreters*). t is compiled into `states :: [STATE]` and the transition function `f :: STATE -> [STATE]`. `Trans` can also be initialized by selecting the constant `Trans` in the current tree and pushing build Trans/TransL. Then f is generated from the axiom `Trans=t` or from axioms for -> and an axiom `states=u` where u simplifies to a list of constants.
- The constant `TransL` is rewritten to the state term `TransL states labels f` by applying an axiom `TransL=t` where t simplifies to a collection of triples consisting of a state s, a label (constant) and the list of direct successors of s (constants; see the matrix interpreter in *Widget interpreters*). t is compiled into `states :: [STATE]`, `labels :: [String]` and the transition function `f :: STATE -> String -> [STATE]`. `TransL` can also be initialized by selecting the constant `TransL` in the current tree and pushing build Trans/TransL. Then f is generated from the axiom `TransL=t` or from axioms for -> and axioms `states=u` and `labels=v` where u and v simplify to lists of constants.

The individual programs are executed as follows:

- **auto**: Given a regular expressions built up from the following grammar, rewrite `auto(e)`.

```
 exp ---> eps | mt | symb | exp*exp | exp+exp | iter(exp) | star(exp) | refl(exp)

symb ---> any string
```

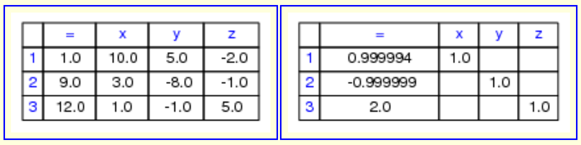


	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4				[],[]						
5		[],[] [6,7],2								
6										
7										
8				[],[]	[],[]					
9				[],[]	[],[]				[],[]	

	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4				[],[]						
5										
6										
7										
8				[],[]	[],[]					
9				[],[]	[],[]				[],[]	

Fig. 10. *A run of* bisim *on the left-hand LTS (see COIN1)*
results in 6 classes of equivalent states: [0], [1], [2], [3,4,8,9], [5], [6], [7].

- **bisim**: Enter an axiom `states=t` and an axiom `TransL=t` or axioms for -> (see above) and rewrite `bisim(TransL)` (see COIN1, COIN2 and CYCLE). Deselect selected subtrees. Simplify the entire displayed tree step by step. Since the simplification steps lead to pictorially representable results (here: triangular matrices), they may be executed from the painter window so that the change of results may be viewed directly in terms their pictorial representations.



	=	x	y	z
1	1.0	10.0	5.0	-2.0
2	9.0	3.0	-8.0	-1.0
3	12.0	1.0	-1.0	5.0

	=	x	y	z
1	0.999994	1.0		
2	-0.999999		1.0	
3	2.0			1.0

Fig. 11. *Two snapshots of a run of* gauss *on the conjunction*
`(10*x)+(5*y)-(2*z) = 1 & (3*x)-(8*y)-z = 9 & x-y+(5*z) = 12`
of linear equations (see gauss1)

- **gauss**: Select the widget interpreter *linear equations* in the *pict type menu*, enter a term of the form `lin(t)` where t is a conjunction of linear equations and simplify `lin(t)` step by step.

Fig. 12. *A run of* nerode *on the left-hand LTS (see auto1) results in 4 classes of equivalent states:*
[1,4], [2,6], [3], [5].

- **nerode**: Enter axioms `states=t` and `labels=u` and an axiom `TransL=t` or axioms for -> (see above) and an axiom `Finals=u` or `FinalsL=u` for the final states resp. (state,label)-pairs of a Moore resp. Mealy automaton. Rewrite `nerode1(TransL,Finals)` or `nerode2(TransL,FinalsL)`, respectively and proceed as in the case of *bisim* (see auto1).

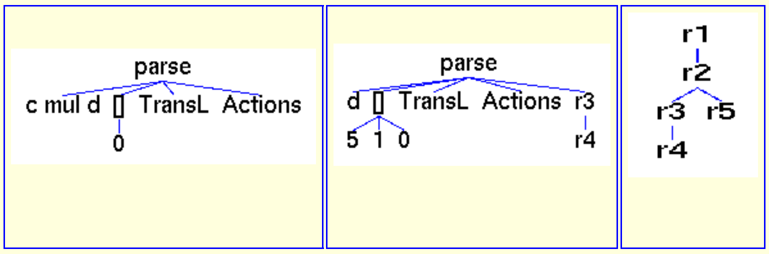


Fig. 13. *Three snapshots of a run of* parse *on lr1*

- **parse**: Enter an axiom `TransL=t` or axioms for -> (see above) and an axiom `Actions=u` for the action table of an LR(1) grammar. Given a sequence `input` of terminal symbols separated by blanks, rewrite `parse(input,[0],TransL,Actions)` and proceed as in the case of *bisim*.

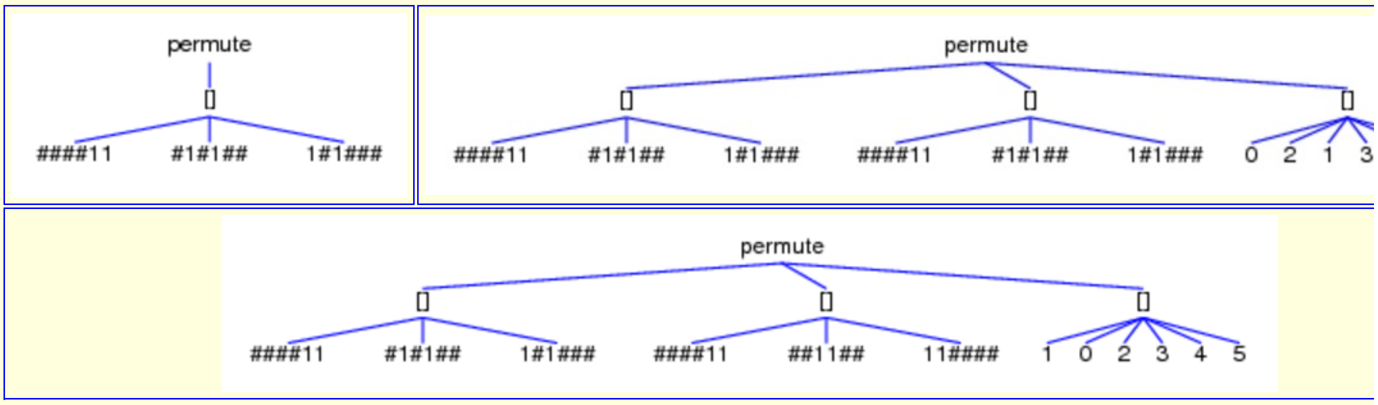
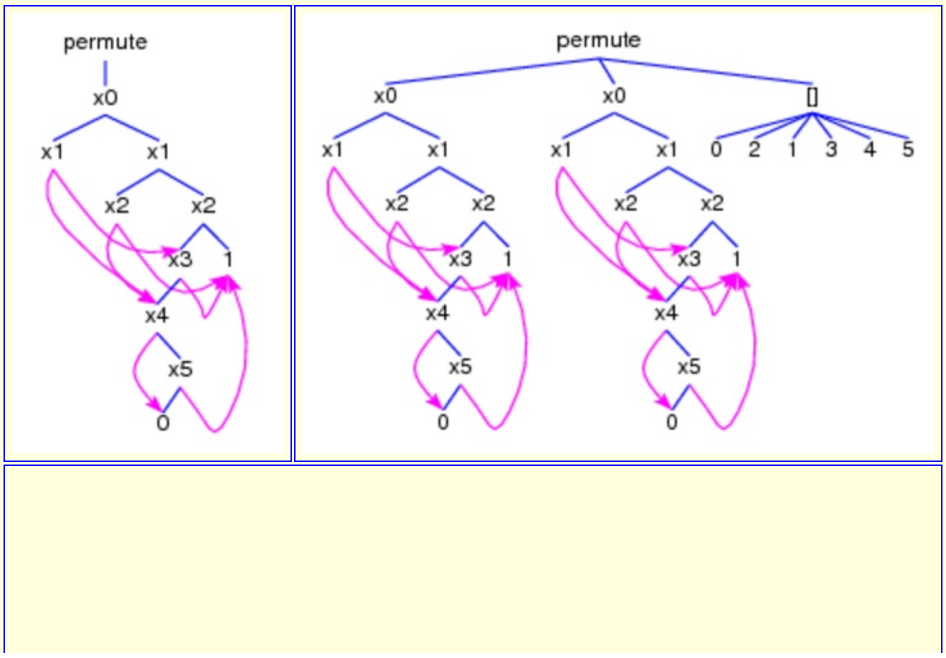


Fig. 14. *Three snapshots of a run of* permute *on DNF4*



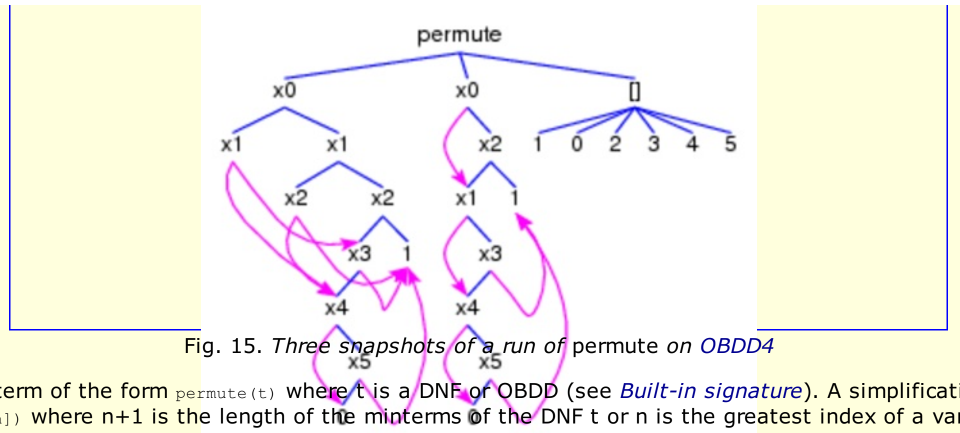


Fig. 15. *Three snapshots of a run of* permute *on OBDD4*

- **permute**: Enter a term of the form `permute(t)` where t is a DNF or OBDD (see *Built-in signature*). A simplification step leads to `permute(t,t,[0,...,n])` where n+1 is the length of the minterms of the DNF t or n is the greatest index of a variable of the OBDD t, respectively. Replace [0,...,n] by a permutation L of this list and perform a further simplification step. It leads to `permute(t,t',L')` where t' is the DNF/OBDD obtained from t by rearranging the variables of t according to L and L' is the successor of L (see *Built-in signature*).

Fig. 16. *Two snapshots of a run of* postflow *on the factorial program:*
x := n; y := 1; while x > 0 & fact(n)=fact(x)*y do y := x*y; x := x-1 od
The invariant fact(n)=fact(x)*y *had to be added to the loop entering condition* x > 0
for ensuring that postflow *terminates*.

- **postflow**: Enter axioms `flow=bool(X1=t1&...&Xn=tn)` for the flowgraph F of an iterative program P and `post=bool(t)` for a postcondition of P. `X1=t1&...&Xn=tn` must be a set of regular equations representing F. t must be a formula over the current signature (see factpost). Enter `postflow(flow,post)`, rewrite `flow` and `post` and simplify the entire displayed tree step by step. The commands and tests of F are colored in green. Simplification steps modify the *assertions* attached to `bool` nodes. In the snapshots shown above, some assertions are hidden behind @. If the simplification has started out from the postcondition `post` of P (the condition at the `out` node) and terminates, the formula at the `in` node is the corresponding precondition, i.e. `pre` implies `post` upon termination of P. The termination of `postflow` cannot be guaranteed! If P involves loops, the loop entering conditions must be generalized to loop invariants (see Fig. 16). Moreover, the stability check of a flowgraph depends on the proof that its current node valuation is equal to the one before the preceding simplification step. But here the nodes are valuated by assertions and thus equality means logical equivalence! Hence it might be necessary to simplify or normalize assertions in order to prove their logical equivalence.

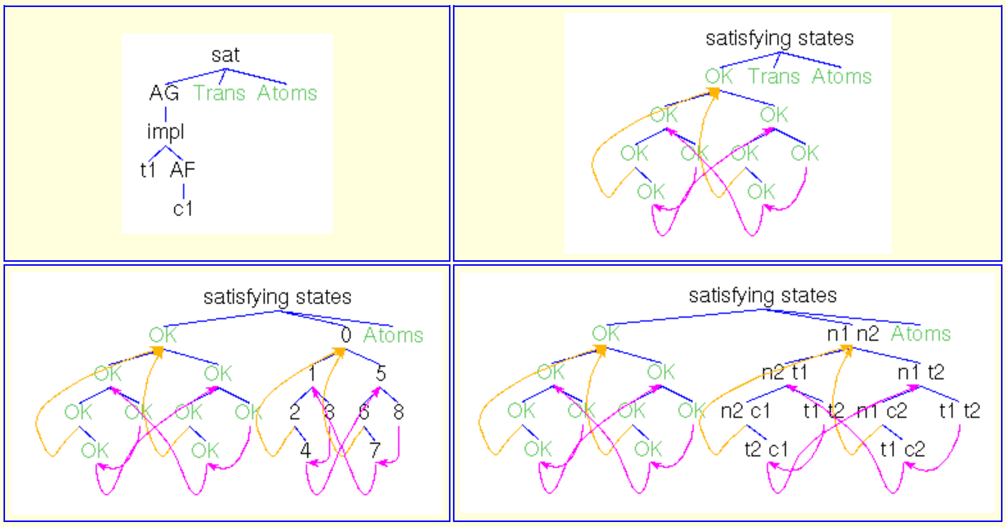
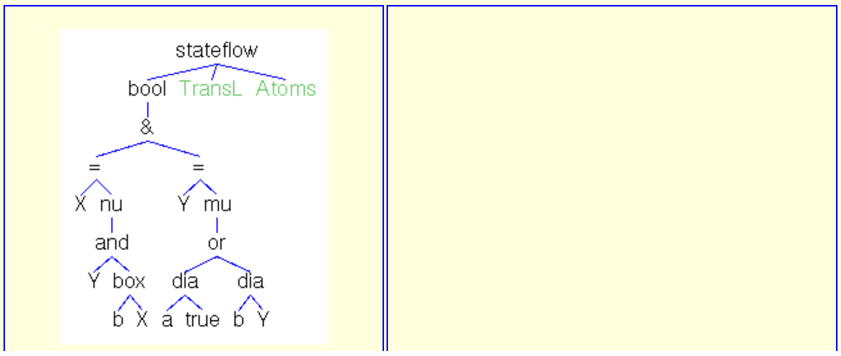


Fig. 17. *Snapshots of a run of* sat *on CTLmutex2*

- **sat**: Enter an axiom `Trans=t` or axioms for -> (see above) and an axiom `Atoms=u` for the labelling of states with atomic formulas. Given a CTL formula `ctl`, rewrite `sat(ctl,Trans,Atoms)` (see CTLmutex1 and CTLmutex2). The result is a term of the form `satisfying states(G,Trans,Atoms)` where G is the transition graph of t. A node N of G is labelled with `OK` if the state N represents satisfies `ctl`. Otherwise N is labelled with `NO` Selecting the root of `Trans` and pressing *build graph* will substitute the original transition graph G' of t for `Trans` (see lower-left picture of Fig. 17). Selecting the root of G' and pressing *label graph* leads to a recoloring of G' such that each node N of G' is labelled with the atomic formulas that the state N represents satisfies (see 4th picture of Fig. 17). The corresponding function is stored in `Atoms` (see above).



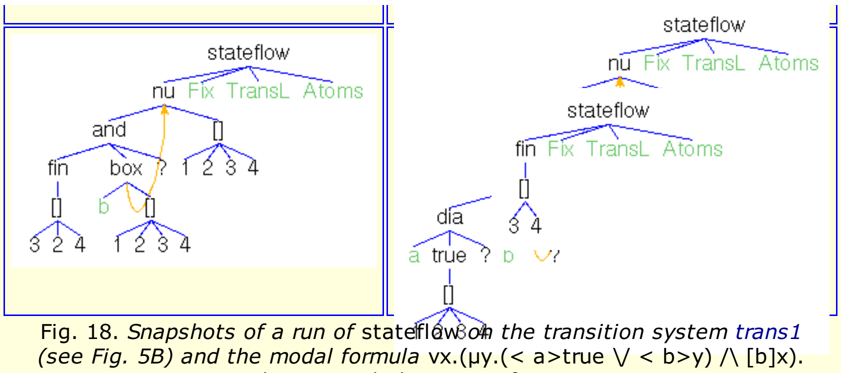


Fig. 18. *Snapshots of a run of* stateflow *on the transition system trans1 (see Fig. 5B) and the modal formula* νx.(μy.(< a>true \/ < b>y) /\ [b]x).
`Fix` stores the minimal alternating fixpoint positions.

- **stateflow**: Enter an axiom `TransL=t` or axioms for -> (see above) and axioms `flow=bool(X1=t1&...&Xn=tn)` for the flowgraph F of a μ-calculus formula and `Atoms=u` for the labelling of certain leaves of F with atomic formulas. `X1=t1,...,Xn=tn` must be regular equations representing F (see trans1 and trans2). Rewrite `stateflow(flow,TransL,Atoms)` and simplify the entire displayed tree step by step. The actions of F are colored in green. Simplification steps modify the state set valuations attached to the nodes of F. Once a fixpoint subformula obtains a simplified value, it is replaced by an `out` node carrying the value. Selecting the root of `TransL` and pressing *build graph* will substitute the transition graph G of t for `TransL` (see 5th picture of Fig. 18). Selecting the root of G and pressing *label graph* leads to a recoloring of G such that each node N of G is labelled with the atomic formulas that the state N represents satisfies. The corresponding function is stored in `Atoms` (see above).

Fig. 19. *Two snapshots of a run of* subsflow *on the factorial program:*
x := n; y := 1; while x > 0 do y := x*y; x := x-1 od

- **subsflow**: Enter an axiom `flow(z1,...,zk)=bool(X1=t1&...&Xn=tn)` for the flowgraph F of an iterative program P. `X1=t1&...&Xn=tn` must be a set of regular equations representing F (see FACTSUBS). Rewrite a term of the form `subsflow(flow(a1,...,ak))` and simplify the reduct step by step. The commands and tests of F are colored in green. Simplification steps modify the program variables' values attached to the nodes of F. In the snapshot above, some of these values are hidden behind @. If F is simplified, the `out` node carries the final values (see [KU]).

Examples

	specification or axioms	**theorems**	**conjectures**	**derivation or proof term**
alignments	ALIGN			
arithmetic and simplifications	NAT WIRTH PRIMS	NATths	EVEN POLY GAUSS1 GAUSS2 GAUSS3 REGEQS SET1 SIMPL SIMPL1 SIMPL2 SIMPL3	ASSOCproof COMMproof DIVproof DIVterm EVproof EVterm EVODproof EXPproof FIBproof POTproof POTterm PRIMSproof WIRTHproof WIRTHterm
binary trees	BTREE REPMIN COBINTREE			REPMINproof REPMINterm MIRRORproof
Boolean functions	BOOL OBDD SWAP			
concept formation	FRUIT	FRUITths	FRUITconjs	FRUIT1proof ... FRUIT4proof
finite sequences	LIST LISTEVAL PARTN LISTREV		FILTER MAP MAP2 SPLIT MERGE SORT FLATTEN ZIP1 ZIP2 ZIP3 ZIP4	PARTproof PARTterm PARTproof2 PARTterm2 SORTproof PERMproof MERGEproof PARTNproof PARTNterm PARTN2proof PARTN2term SPLITNproof LISTREVproof
imperative programs	FACTPOST FACTSUBS		PROG1 PROG2 PROG3 PROG4	
infinite sequences	STREAM ABP FIBEQ	STREAMths	STREAMconjs	FAIRBLINK proof term BLINKZIP proof term ODDSZIP EVENSZIP ITER1ITER2 proof term ITERLOOP proof term ODDSEVENS proof term MAPFACT proof term MAPITER1 proof term MAPLOOP proof term MAPLOOP0 proof term NATLOOP proof term INVINV MORSE ZIPODDS proof term

				ABPproofA1 ABPproofA2 ABPproofB ABPproofC ABPproofD ABPproofE ABPproofF
needed narrowing	NEED			NEEDproof NEEDterm NEED1proof NEED2proof
parser	LR1 LR2			LR1run LR2run
permutations and partitions	LOG		LOGproof LOGproof1 LOGproof2 PARTsols	
pictures	CARPET NICETREE PYTREE		files whose names start with a small letter	
regular expressions	COREGS TRANS0		CORproof	
sets and relations	SET GRAPHS RELALG RELALGDB RELALGP NEWMAN	SET1	SETproof RELALG1proof RELALG2proof RELALGP1proof RELALGP1term RELALGP2proof RELALGP2term NEWMANproof NEWMANterm NEWMAN2proof NEWMAN2term	
stacks	STACK STACKIMPL STACKIMPL2			TOPEMPTYproof TOPEMPTYterm TOPPUSHproof TOPPUSHterm POPEMPTYproof POPEMPTYterm POPPUSHproof POPPUSHterm PUSHCOMPproof PUSHCOMPterm UPDproof UPDterm
transition systems and model checking	ACCOUNT AUTO1 COIN1 COIN2 BOTTLE BOTTLEAC CTL CTLlab CTLlist CTLlablist CTLMUTEX1 CTLMUTEX2 CYCLE ECHO ECHOAC HANOI KNIGHT LTL MUTEX PHIL PHILAC PUZZLE QUEENS ROBOT ROBOTacts TRANS0 TRANS1 TRANS2			ACCOUNTsol BOTTLEsols CTLproof CTLlabproof CTLlabterm CTLlablistproof ECHOproof ECHOACproof KNIGHTsols MUTEXproof PHIL1proof PHIL2proof PHILAC1proof PHILAC2proof PUZZLEproof ROBOTsol ROBOTsols ROBOTsols2 ROBOTactsproof

Widget interpreters

Built on top of the Tk interface module Tk.hs, the module *Epaint* provides features for creating and editing pictorial term representations. These are displayed in the painter window of a solver when a *paint button* is pushed. The scroll region of this window is adapted automatically to the displayed picture.

Expander2 provides several widget interpreters and combinators thereof, which recognize paintable terms and transform them into pictorial representations. The actual widget interpreter can be selected from the *pict type* menu. The default interpreter is *matrices*. The alignment enumerator and the palindrome enumerator are associated with the *alignment* interpreter, the dissection enumerator with *rectangles* and the partition enumerator with *partition*.

The basic elements of pictorial term representations are called **widgets**. A **picture** is a list of widgets. A **widget graph** G is a pair consisting of a picture [w1,...,wn] and a list [as1,...,asn] of sublists of [1,...,n] that represents the set A of arcs of G:

A = {(wi,wj) | j ∈ asi, 1 <= i,j <= n}.

For editing widget graphs, see Figures 20 and 21 and the *paint buttons*.

A graph is saved in Haskell code to *Examples/file* or in eps format to *Pics/file.eps* by writing *file* resp. *file.eps* into the *save to* field and pushing the *Down* key while the cursor is in this field.

Widgets are encoded in Haskell as follows:

```
 data Widget_ = Arc Color ArcStyleType Point Float (Float,Float) |

Arc0 State ArcStyleType Float Float |

 Arc0, Path0 and Tree0 are abstract versions of Arc, Path resp. Tree

which they are turned into before being displayed.

Bunch Widget_ [Int] |

 Bunch w ns represents widget w together with arcs leading from w

to the widgets at positions ns.

Circ State Float | CircA State Float | Dot Color Point |

 CircA and RectA ignore the scale of enclosing turtles.

Fast Widget_ | File_ String | Gif String Point | New | Old | Path Color Int [Point] |
Path0 State Int [Point] | Poly State Int [Float] Float | Rect State Float Float |

RectA State Float Float | Repeat Widget_ | Snow State Int Float | Text_ State [String] |

Tree Color Color (Term TNode) | Tree0 State String Color [Term TNode] |

Tria State Float | Turtle State Float [TurtleAct] | White


data TurtleAct = Move Float | MoveA Float | Jump Float | JumpA Float | Turn Float |

MoveA and JumpA ignore the scale of the enclosing turtle.

Open Color Int | Scale Float | Close | Draw | Widg Widget_ | WidgB Widget_

 Widg w ignores the orientation of the enclosing turtle,

WidgB w adds it to the orientation of w.


type State = (Point,Float,Color,Int)

type TNode = (String,Point)

type Point = (Float,Float)
```

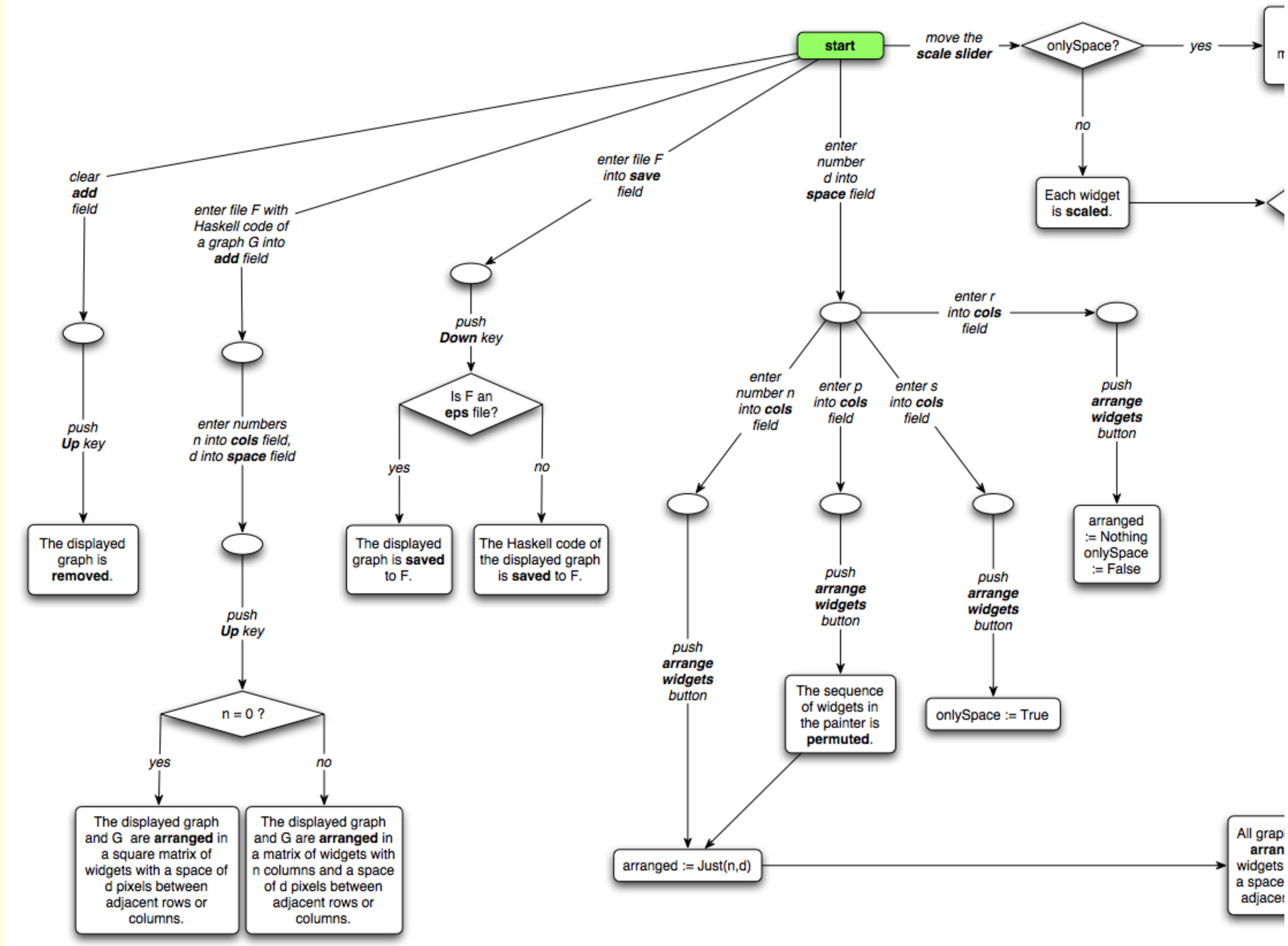
A widget is a two-dimensional object with state. The state is a quadruple (p,a,c,i) consisting of the widget's actual position (as Cartesian coordinates), orientation (in degrees), (brilliant) color and lightness (see the picture operators *newLight* and *nextLight* described below). Only color and lightness can be set directly by the user. The standard values of p and a are (0,0) and 0, respectively. Other values are computed by the painter in dependence of picture (generating or modifying) operators called by the user.

Certain picture operators interpreted by *polygon* (see below) have a color parameter c that is used as the first element of a list cs of **equidistant colors**, i.e. the difference between the RGB values of two subsequent colors of cs is always the same. The colors are ordered like in a rainbow.For ensuring the correct computation of cs from c, c should be a **pure (hue) color**, i.e. c is neither black nor white and at most one of the R-, G- and B-values of c is different from both 0 and 255. If you want the picture operator to create a light or dark version of the colors of cs, you must apply it to the pure version of c and then apply to the result the picture operator *newLight* or *nextLight* with the desired lightness value i (see below). Then all colors of cs will be lightened (or darkened if i is negative) as desired.

Colors must be entered as strings generated by the color grammar (see the *Grammar*).

Each widget interpreter is a Haskell function of type `Term String -> Maybe Picture`.

A displayed graph is always aligned to the top and the left of the painter's canvas. The available graph editing commands are shown in Figures 20 and 21.



start
move the scale slider
onlySpace?
yes
no
Each widget is scaled.
clear add field
enter file F with Haskell code of a graph G into add field
enter file F into save field
enter number d into space field
push Up key
The displayed graph is removed.
enter numbers n into cols field, d into space field
push Up key
n = 0 ?
yes
no
The displayed graph and G are arranged in a square matrix of widgets with a space of d pixels between adjacent rows or columns.
The displayed graph and G are arranged in a matrix of widgets with n columns and a space of d pixels between adjacent rows or columns.
push Down key
Is F an eps file?
yes
no
The displayed graph is saved to F.
The Haskell code of the displayed graph is saved to F.
enter r into cols field
enter number n into cols field
enter p into cols field
enter s into cols field
push arrange widgets button
arranged := Nothing onlySpace := False
push arrange widgets button
push arrange widgets button
push arrange widgets button
The sequence of widgets in the painter is permuted.
onlySpace := True
arranged := Just(n,d)

Fig. 20. *Graph editing actions I*

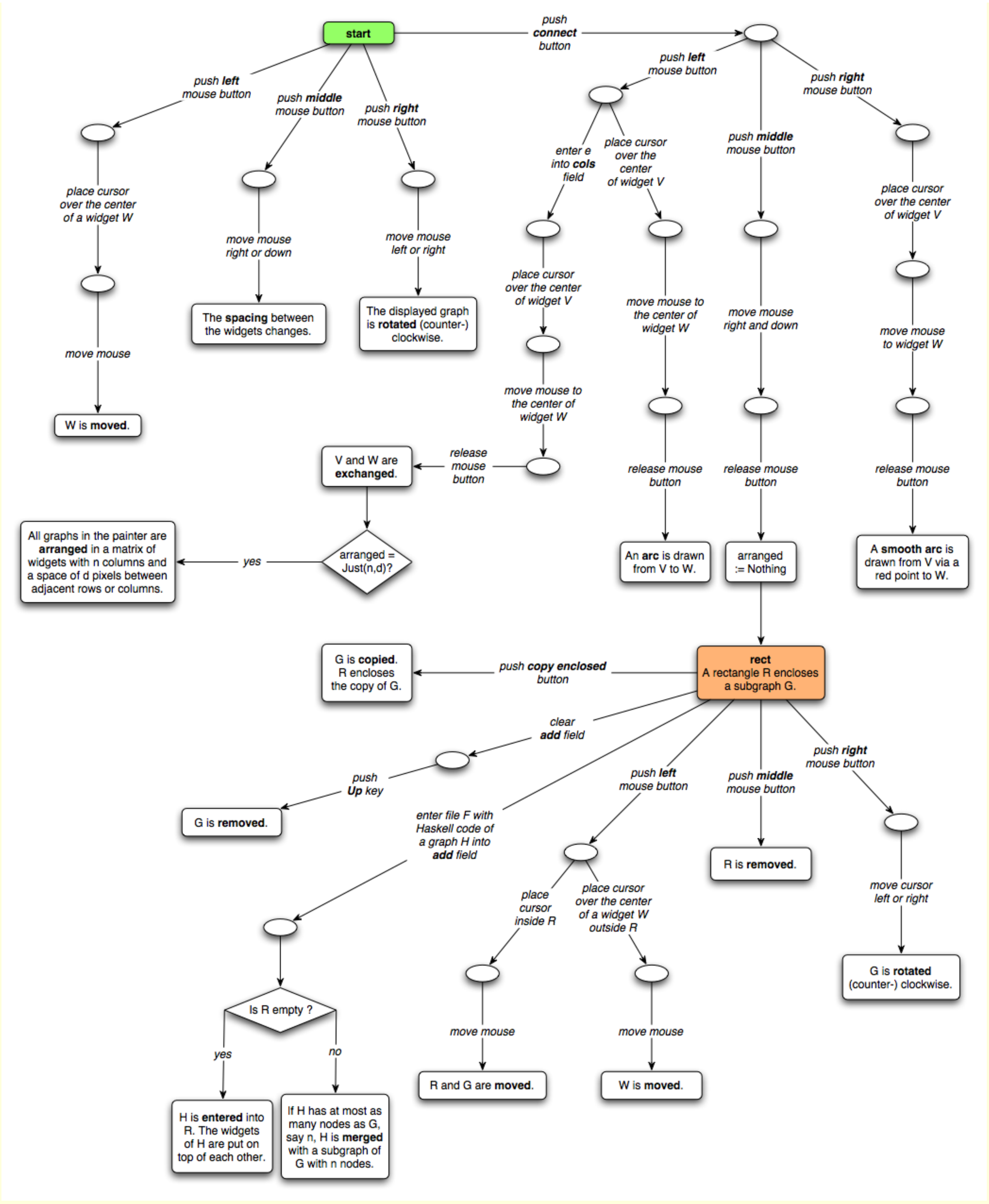

start
push connect button
push left mouse button
push middle mouse button
push right mouse button
place cursor over the center of a widget W
move mouse
W is moved.
move mouse right or down
The spacing between the widgets changes.
move mouse left or right
The displayed graph is rotated (counter-) clockwise.
push left mouse button
enter e into cols field
place cursor over the center of widget V
place cursor over the center of widget V
move mouse to the center of widget W
release mouse button
V and W are exchanged.
arranged = Just(n,d)?
yes
All graphs in the painter are arranged in a matrix of widgets with n columns and a space of d pixels between adjacent rows or columns.
move mouse to the center of widget W
release mouse button
An arc is drawn from V to W.
push middle mouse button
move mouse right and down
release mouse button
arranged := Nothing
push right mouse button
place cursor over the center of widget V
move mouse to widget W
release mouse button
A smooth arc is drawn from V via a red point to W.
rect
A rectangle R encloses a subgraph G.
push copy enclosed button
G is copied. R encloses the copy of G.
clear add field
push Up key
G is removed.
enter file F with Haskell code of a graph H into add field
Is R empty ?
yes
no
H is entered into R. The widgets of H are put on top of each other.
If H has at most as many nodes as G, say n, H is merged with a subgraph of G with n nodes.
push left mouse button
place cursor inside R
move mouse
R and G are moved.
place cursor over the center of a widget W outside R
move mouse
W is moved.
push middle mouse button
R is removed.
push right mouse button
move cursor left or right
G is rotated (counter-) clockwise.

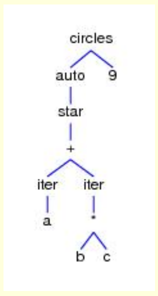
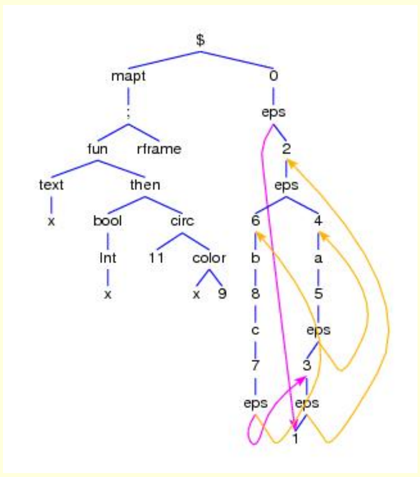
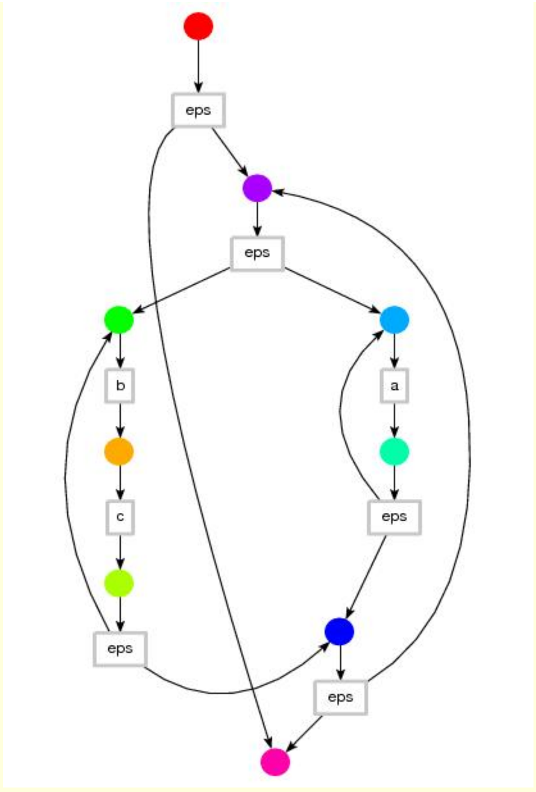
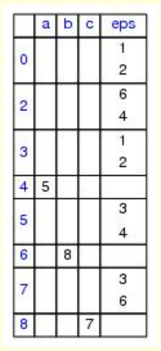
Fig. 21. *Graph editing actions II*
*In all states reachable from **rect**, the displayed graph contains a rectangle whose enclosed widgets are processed differently from the rest of the graph.*

pict type menu

The actual widget interpreter is set in the *pict type menu*. The interpreters, the term patterns they recognize and the widget graphs they display on the canvas of a painter read as follows.

alignment recognizes syntax trees generated by the grammar G1 or G2 of section *Alignments and palindromes* are displayed as horizontal alignments.

graph transforms the current tree into a widget graph. Subterms of the form widg(t1,...,tn,t) such that *polygon* (see below) recognizes t as a single widget w is turned into the subgraph w(g1,...,gn) where gi is the widget graph for ti, 1<=i<=n. Other non-pointer nodes are turned into text widgets. Each non-tree edge of the current tree is compiled into a B-splined arc with a red control point for interactive reshaping.

			
t1	t2	p	w

circles
auto
9
star
+
iter
iter
a
*
b
c

$
mapt
0
:
eps
fun
rframe
2
text
then
eps
x
bool
circ
6
4
Int
11
color
b
a
x
x
9
8
5
c
eps
7
3
eps
eps
1

eps
eps
b
a
c
eps
eps
eps

	a	b	c	eps
0				1 2
2				6 4
3				1 2
4	5			
5				3 4
6		8		
7				3 6
8			7	

Fig. 22. *Simplifying and rewriting term t1 with axioms of trans0*
leads to term t2, which is further simplified to a term that is compiled by **graph** *into picture p and by* **matrix** *into widget w.*

linear equations: A term of the form `p1=r1 & ... & pn=rn` where p1,...,pn are polynomials and r1,...,rn are real numbers is interpreted as a system of linear equations and displayed as the corresponding matrix of coefficients. The variables occurring in the equations must be part of the current signature (see *gauss1*).

matrix interprets the following rooted graphs and displays them as corresponding matrices.

- Rooted graphs generated by the following grammar G1 are regarded as labelled transition systems.

```
 LTS ---> stateGraph | stateGraph <+> LTS

stateGraph ---> state(labelGraph) | position of a state node

labelGraph ---> label(rooted)

state ---> string

label ---> string
```

- Rooted graphs generated by the following grammar G2, but not by G1, are regarded as unlabelled transition systems.

```
 TS ---> stateGraph | stateGraph <+> TS

rooted ---> state(stateGraph) | position of a state node

state ---> string
```

- A term of the form `["b1",...,"bn"]` where b1,...,bn are words over {0,1,#} of the same length is interpreted as a DNF and displayed as the equivalent Karnaugh diagram (see *Built-in signature*).
- Haskell objects of the form `Matrix f ss` and `MatrixL f ss` are displayed as triangular matrices with a row and a column for each element of `ss`. Symmetric relations represented by triangular matrices are generated when, e.g., a `bisim` or `nerode` term is simplified (see *Simplifications*).
- Given constants x11,...,x1m1,...,xk1,...,xkmk,y11,...,y1n1,...,yk1,...,yknk, a collection (see *Built-in signature*)

  ```
   C(([x11,...,x1m1],[y11,...,y1n1]),...,([xk1,...,xkmk],[yk1,...,yknk]))
  ```

  is displayed as a matrix representing the Boolean function that maps (xir,yis), 1 <= i <= k, 1 <= r <= mi, 1 <= s <= ni, to True and all other pairs to False.
- Given constants x11,...,x1m1,...,xk1,...,xkmk,y11,...,y1n1,...,yk1,...,yknk and lists L1,...,Lk of terms, a collection

  ```
   C(([x11,...,x1m1],[y11,...,y1n1],L1),...,([xk1,...,xkmk],[yk1,...,yknk],Lk))
  ```

  is displayed as a matrix representing the partial function f that maps (xir,yjs), 1 <= i <= k, 1 <= r <= mi, 1 <= s <= ni, to Li. The elements of Li are written vertically below each other. Moreover, a triple (x,"else",L) in the graph of f is interpreted as the set of all triples (x,y,L) such that y =/= "else", (z,y,us) is in the domain of f for some z and us, but (x,y,us) is not in the domain of f for all us.
- Any other collection or tuple

  ```
   C(f1(t11,...,t1n1),...,fk(tk1,...,tknk))
  ```

  is displayed as the list of the lists [f1,t11,...,t1n1],...,[fk,tk1,...,tknk]. For all 1 <= i <= k, fi is colored red and the elements of the list [f1,t11,...,t1n1] are written vertically.

matrix solution recognizes each solved formula

```
 Any Z1:x1=t1 &...& Any Zk:xk=tk & All Z(k+1):x(k+1)=/=t(k+1) &...& All Zn:xn=/=tn
```

and submits the terms t1,...,tn to *matrix*.

matrices combines the maximal subgraphs of the current graphs that are recognizable by *matrix*.

partition interprets the displayed tree t as a nested partition (see *Dissections and partitions*) and draws t as a square combined of colored rectangles each of which represents a leaf of t. The coloring method can be changed by pushing the *combis* button (see the *paint buttons*).

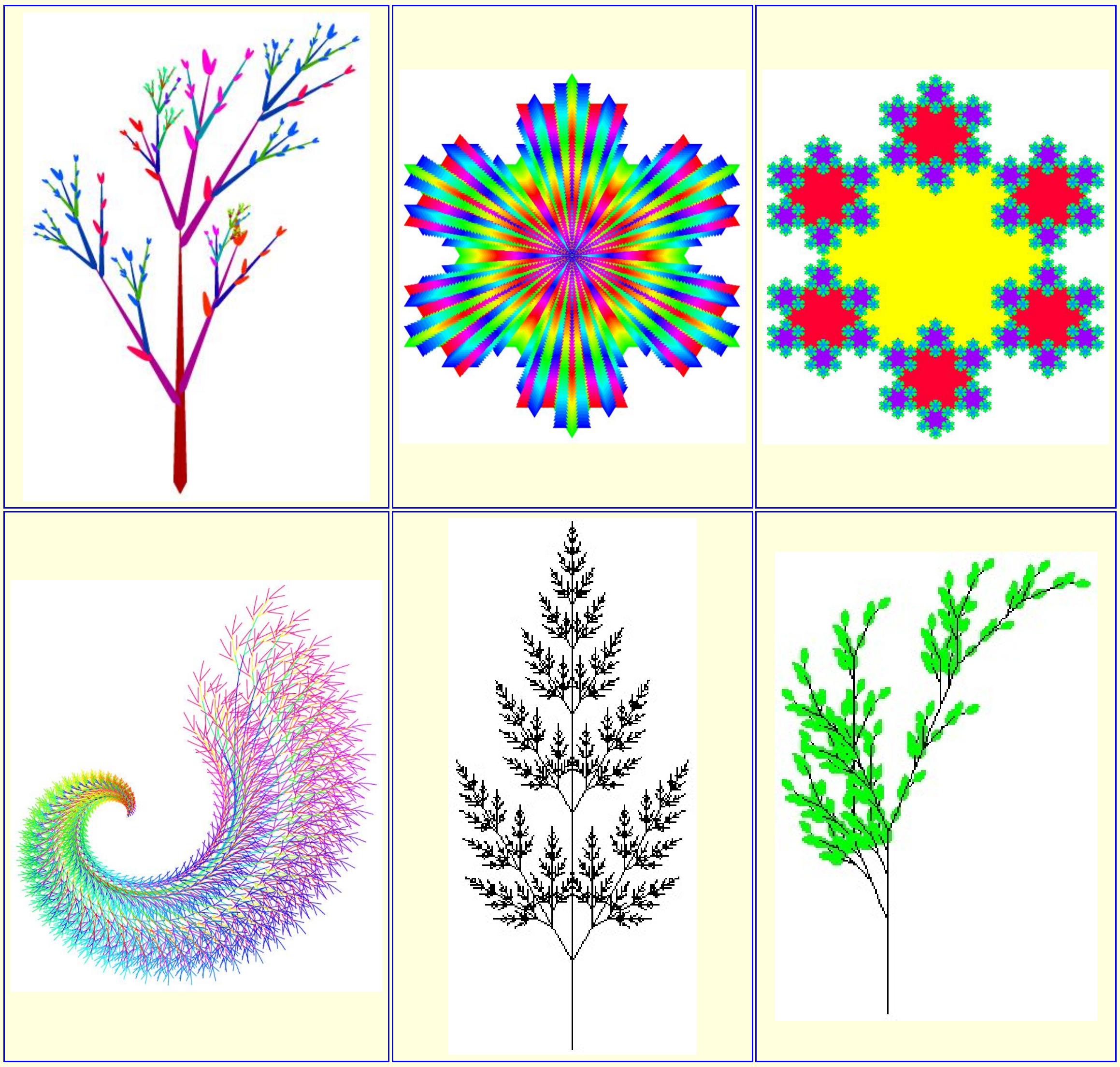
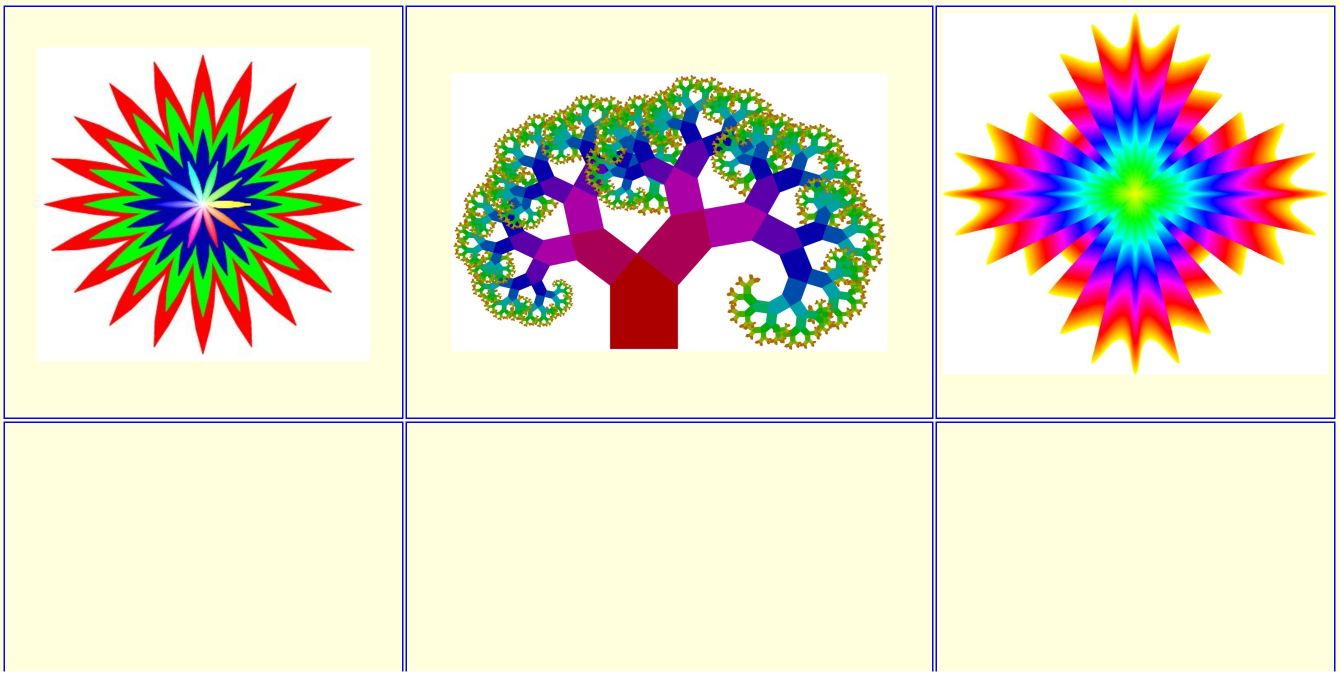


Fig. 23. *Widgets drawn by the polygons interpreter*



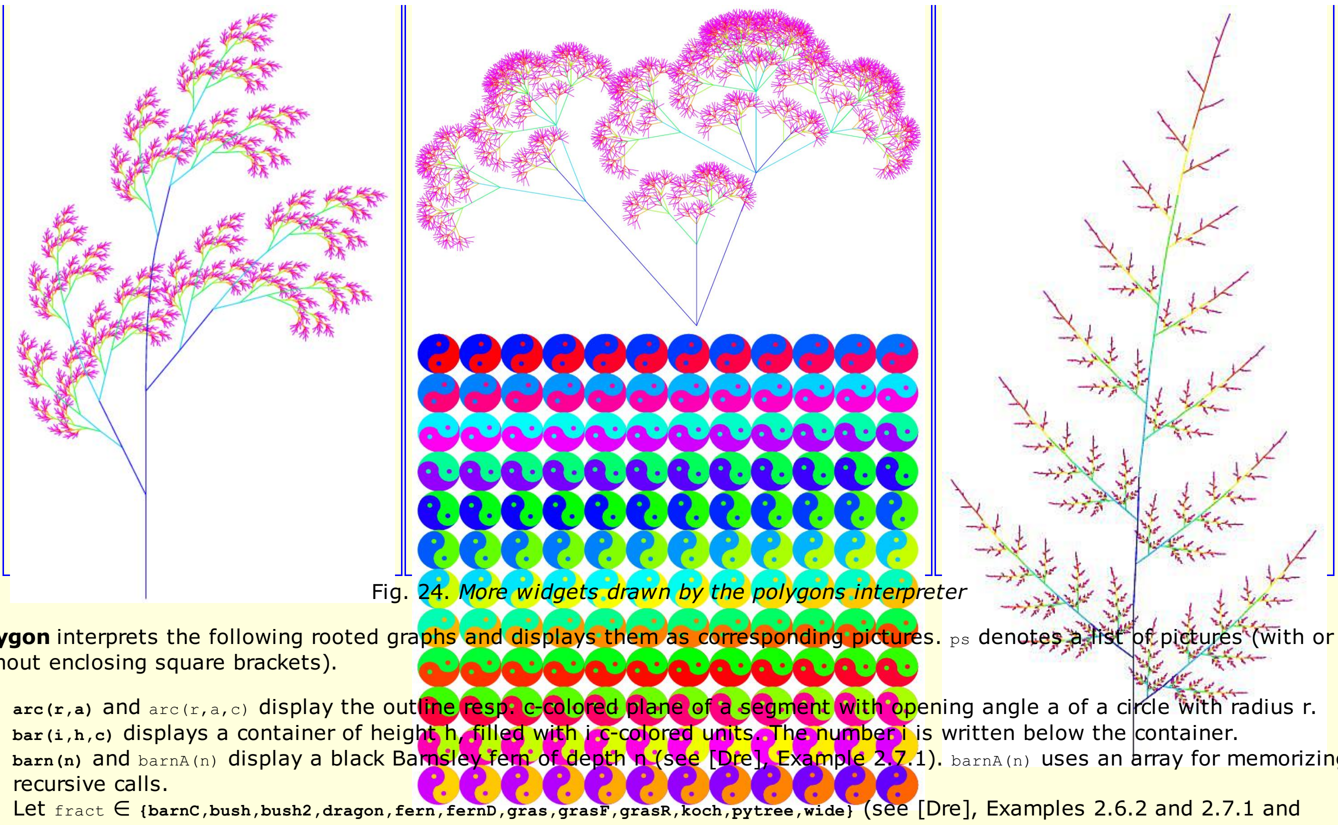


Fig. 24. *More widgets drawn by the polygons interpreter*

polygon interprets the following rooted graphs and displays them as corresponding pictures. `ps` denotes a list of pictures (with or without enclosing square brackets).

- **`arc(r,a)`** and `arc(r,a,c)` display the outline resp. c-colored plane of a segment with opening angle a of a circle with radius r.
- **`bar(i,h,c)`** displays a container of height h, filled with i c-colored units. The number i is written below the container.
- **`barn(n)`** and `barnA(n)` display a black Barnsley fern of depth n (see [Dre], Example 2.7.1). `barnA(n)` uses an array for memorizing recursive calls.
- Let `fract` ∈ **`{barnC,bush,bush2,dragon,fern,fernD,gras,grasF,grasR,koch,pytree,wide}`** (see [Dre], Examples 2.6.2 and 2.7.1 and Figures 2.30 and 2.32).
  `fract(n)` and `fract(n,c)` display the respective fractal of depth n. The first level of the fractal is colored in black resp. c. If the k-th level is colored in c, then the (k+1)-th level is colored with the successor of c with respect to a circular list of n equidistant colors (see above).
- **`blos(n,d,c)`** displays the c-colored outline of a blossom with n leaves built up from lines of length d.
- **`blosF(n,h,d,c)`** displays a c-colored (*filled*) blossom with n leaves of height h and width d <= h.
- **`blosR(n,h,d)`** displays a blossom b with n leaves of height h and width d. For each two adjacent leaves l and l', the color of l is the successor of the color of l' with respect to a circular list of n equidistant colors.
- **`blosS(n,d,a,c)`** displays a c-colored blossom with n leaves built up from smooth lines of length d and angle a at the vertex of two adjacent lines.
- **`circ(r)`** and **`circ(r,c)`** display the outline resp. c-colored plane of a circle with radius r.
- **`clear`** clears the canvas.
- **`colbars(c)`** represents a color `RGB r g b` with r,g,b >= 0 as three containers of height 127, filled with r, g resp. b r-, g- resp. b-colored units. The numbers r/2,g/2 and b/2 are written below the respective container.
- **`dark(ps)`** displays `light(-14,ps)` (see below).
- **`fadeB(w)`** and **`fadeW(w)`** return the picture consisting of copies of widget w that fade to *black* resp. *white* in 42 steps and then back to the original lightness of w.
- **`fast(w)`** provokes the fast painting of widget w.
- **`fern2(n,d,r)`** and **`fern2(n,d,r,c)`** display a black resp. c-colored fern fractal of depth n, apical delay d, internode elongation rate r (see [Pru], Section 5.3).
- **`flash(w)`** returns the picture consisting of 102 copies of widget w each of which is colored with the successor of the color of its predecessor with respect to a circular list of 102 equidistant colors.
- **`flipH(ps)`** and **`flipV(ps)`** flip the pictures of ps *horizontally* resp. *vertically*.
- **`gif(F)`** displays the contents of file F.gif.
- **`grow(ps)`** places the first two widgets v and w of the pictures of ps on the two upper sides of a `trunk` (see below). Each term constructed from `trunk` and `grow` is displayed as a Pythagorean tree t. For each two adjacent branches b and b', the color of b is the successor of the color of b' with respect to a circular list of height(t) equidistant colors.
- **`grow5(n,ps)`** places the first five widgets of the pictures of ps at the *five* leaf positions of a branch `rhomb5(n)` (see below). Each term constructed from `rhomb5(n)`, `grow5`, `growR` (see below) and any leaf widget is displayed as a tree t.
- **`growR(n,ps)`** builds the same tree t as `grow5(n,ps)` does, except for the coloring of the branches of t. For each two adjacent branches b and b', the color of b is the successor of the color of b' with respect to a circular list of height(t) equidistant colors.
- **`hdots(ps)`** displays the vertices of the outlines of the widgets of the pictures of ps in dark versions of the colors of the respective widgets.
- **`hframe(ps)`** displays the outlines of the widgets of the pictures of ps in dark versions of the colors of the respective widgets.
- **`hframe2(ps)`** displays the widgets of the pictures of ps and their outlines in dark versions of the colors of the respective widgets.
- **`hilb(n)`** displays a smooth Hilbert curve of depth n.
- **`hilb(n,c)`** displays a Hilbert curve of depth n. The first level is c-colored. If the k-th level is d-colored, then the (k+1)-th level is colored with the successor of d with respect to a circular list of n equidistant colors.
- **`leaf/S(d,a)`** displays `leaf/S(d,a,green)`.
- **`leaf(d,a,c)`** displays a c-colored leaf built up from lines of length d and angle a at the vertex of two adjacent lines.
- **`leafF(h,d)`** displays `leafF(h,d,green)`.
- **`leafF(h,d,c)`** displays a c-colored (*filled*) leaf of height h and width d <= h.
- **`leafS(d,a)`** displays `leafS(d,a,green)`.
- **`leafS(d,a,c)`** displays a c-colored leaf built up from smooth lines of length d and angle a at the vertex of two adjacent lines.
- **`light(ps)`** displays `light(21,ps)`.
- **`matrix(t)`** displays the widget obtained by applying *matrix* to t (see above).
- **`meet(n,ps)`** displays the (n+6)-th representation of ps (see *combis* in the *paint buttons* section).
- **`new`** creates a new picture scanner that will process subsequent widgets and run in parallel to the running scanners. The new scanner is put on top of the stack of all running scanners.
- **`newLight(i,n,ps)`** modifies the lightness of the pictures of ps. The range of lightness values starting from black and ending with

white is divided into n intervals of equal length. The lightness corresponding to the lower bound of the i-th interval is assigned to all pictures of ps.

- **`nextLight(i,ps)`** also modifies the lightness of the pictures of ps. Here the range of lightness values is divided into 84 intervals of equal length. The lightness corresponding to the lower bound of the k-th interval is represented in the state of a widget w by l(w)=k-42. In terms of the color c of w, l(w)=0 represents the most brilliant version of c, positive values of l(w) light and negative values dark versions of c. For all widgets w of ps, nextLight(i,ps) increases l(w) by i. If i is positive, ps becomes lighter, otherwise ps becomes darker.
- **`old`** pops the stack of running scanners by one element. Hence subsequent widgets will be processed by the new stack top.
- **`osciL(h,c)`** oscillates the height of a c-colored *leaf* (`leafF`; see above) between 1 and h pixels.
- **`osciP(n,d,c,c')`** oscillates the slope angles of a c- resp. c'-colored smooth *plait* (see below) with n peaks and slope length d between 1 and 85 degrees.
- **`osciW(n,d,c)`** oscillates the slope angle of a c-colored smooth *wave* (see below) with n peaks and slope length d between 1 and 85 degrees.
- **`outline(ps)`** displays the outlines of the widgets of the pictures of ps in dark versions of the colors of the respective widgets.
- **`path/S/F/SF[ps]`** displays `path/S/F/SF([ps],black)`.
- **`path([(x1,y1),...,(xn,yn)],c)`** displays a c-colored path with vertices (0,0),(x2-x1,y2-y1),...,(xn-x1,yn-y1) and rotation point (0,0).
- **`pathS(ps,c)`** computes the value of `path(ps,c)` and displays a B-splined version of the computed path.
- **`pathF([(x1,y1),...,(xn,yn)],c)`** displays a c-colored (*filled*) polygon with vertices (0,0),(x2-x1,y2-y1),...,(xn-x1,yn-y1) and rotation point (0,0). *Since the Tk interface does not cope with lines or polygons with more than 100 vertices, they are split into smaller ones before being drawn. Unfortunately, this may lead to additional vertices of smooth lines or additional edges of filled polygons!*
- **`pathSF(ps,c)`** displays a smooth version of `pathF(ps,c)`.
- **`peaks/R(w)`** displays `peaks/R(w,33)`.
- **`peaks(w,m)`** and **`peaksR(w,m)`** changes the number n of peaks of a polygon w. Starting out from n, the number of peaks is first increased up to m'=min(m,33) (if w has an odd number of vertices) or m'=min(m,50) (if w has an even number of vertices), then decreased down to three (if w has an odd number of vertices) or two (if w has an even number of vertices) and, finally, increased again up to n. `peaksR` changes the color of w such that each copy of w is colored with the successor of the color of its predecessor with respect to a circular list of m'-1 equidistant colors.
- **`pie(n,r)`** displays a pie with n pieces and radius r. For each two adjacent pieces p and p', the color of p is the successor of the color of p' with respect to a circular list of n equidistant colors.

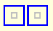


Fig. 24. *The two solutions of* `loop(7,[(0,0)],ps)`
obtained from applying axioms of BOTTLE *(cf. [P01], Section 3.3)*

- **`pile(i,h)`** displays a container, consisting of h squares filled with i light blue units.
- **`place(w,(x1,y1),...,(xn,yn))`** and **`place(w,[(x1,y1),...,(xn,yn)])`** display widget w at positions (x1,y1),...,(xn,yn).
- **`plait/S(n,d,a,c)`** displays `wave/S(n,d,a,c)`. Two widgets `plait/S(n,d,a,c)` and `plait/S(n,d,-a,c)` are drawn on the same baseline.
- **`poly/S/F/SF(k,[rs])`** displays `poly/S/F/SF(k,[rs],black)`.
- **`poly(k,[r1,...,rn],c)`** and **`polyF(k,[r1,...,rn],c)`** displays the c-colored outline resp. c-colored (*filled*) plane of a polygon with m=k*n' vertices p1i1,...,p1in',...,pki1,...,pkin' where [ri1,...,rin'] is the sublist of nonzero elements of [r1,...,rn] and for all 1 <= i <= k and all i1 <= j <= in', (rj,((i-1)*n+j)*360/m) are the polar coordinates of pij. *If m > 100, the polygon is decomposed with possibly undesired effects (see* `pathF`*)!*
- **`polyR(k,[r1,...,rn])`** and **`polyRS(k,[r1,...,rn])`** display a colored polygon with k*n' vertices as in the case of `poly`. The polygon consists of k*n triangles such that one vertex of each triangle coincides with the anchor of the polygon. For each two adjacent triangles t and t', the color of t is the successor of the color of t' with respect to a circular list of k*n equidistant colors. `polyRS` displays smooth versions of the triangles in a way that the anchor vertex of each triangle remains at its original place.
- **`polyS(k,[r1,...,rn],c)`** and **`polySF(k,[r1,...,rn],c)`** display smooth versions of poly(k,[r1,...,rn]) and `polyF(k,[r1,...,rn],c)`, respectively. *If k*n > 99, the polygon is decomposed with possibly undesired effects (see* `pathF`*)!*
- **`pulse(w)`** displays the turtle widget consisting of 20 copies of widget w each of which is smaller than its predecessor. Then, again in 20 steps, w is enlarged up to the original size.
- **`rainbow(w)`** displays `rainbow(w,102,0,0,2)`.
- **`rainbow(w,n)`** displays `rainbow(w,n,0,0,2)`.
- **`rainbow(w,n,d)`** displays `rainbow(w,n,d,360,2)`.
- **`rainbow(w,n,d,a)`** displays `rainbow(w,n,d,a,2)`.
- **`rainbow(w,n,d,a,sc)`** displays n copies of widget w each of which is colored differently from its predecessor. For each two successive copies w1 and w2 of w, the color of w1 is the successor of the color of w2 with respect to a circular list of n equidistant colors. Moreover, copy i of w is the result of scaling down w by the factor c, turning w by an angle of b degrees and moving it by d pixels where b=i*360/n if a >= 360 and b=a otherwise and where c=(n-i)/n if sc > 1 and c=sc otherwise.
- **`rainbow2(w)`** displays `rainbow2(w,102,0,0,2)`.
- **`rainbow2(w,n)`** displays `rainbow2(w,n,0,0,2)`.
- **`rainbow2(w,n,d)`** displays `rainbow2(w,n,d,360,2)`.
- **`rainbow2(w,n,d,a)`** displays `rainbow2(w,n,d,a,2)`.
- **`rainbow2(w,n,d,a,sc)`** displays `rainbow(w,n,d,a,sc)` except that the list of equidistant colors is permuted such that the successor of a color c is most distant from c.
- **`rect(b,h)`** and **`rect(b,h,c)`** display the outline resp. c-colored plane of a rectangle with breadth 2*b and height 2*h.
- **`repeat(ps)`** repeats the display of the pictures of ps until the *stop* or *back to Solver1/2* button is invoked (see the *paint buttons*).
- **`reverse(ps)`** displays the reversal of the concatenation of the pictures of ps.
- **`rframe(ps)`** displays a rectangular frame around each widget of the pictures of ps, colored with the respective complements of the colors of the widgets.
- **`rhomb`** displays `rhomb(green)`
- **`rhomb(c)`** displays a c-colored leaf.
- **`rhomb5(n)`** displays a brown branch with *five* green leaves. The number n indicates where the individual leaves are placed at the branch (see [Dre], Example 3.3.3).
- **`rotate(w)`** displays `rotate(w,10)`.
- **`rotate(w,a)`** displays 360/a copies of widget w. copy i+1 is the result of turning copy i by an angle of a degrees. copy i is painted white before copy i+1 is drawn.
- **`rotateC(w)`** displays `rotateC(w,10)`.
- **`rotateC(w,a)`** displays 360/a copies of widget w. copy i+1 is the result of turning copy i by an angle of a degrees. The canvas is *cleared* before copy i+1 is drawn.
- **`shineB/W(w)`** displays `shineB/W(w,0,0)`.
- **`shineB(w,d)`** and **`shineW(w,d)`** display 43 copies of widget w each of which is lightened darker (*blacker*) resp. lighter (*whiter*) than

its predecessor. Moreover, copy i of w is the result of scaling down w by the factor (42-i)/42, turning it by an angle of i degrees and moving it by d pixels.

- **shineB(w,d,a)** and **shineW(w,d,a)** work the same as `shineB(w,d)` resp. `shineW(w,d)` except that all copies of w are turned by a constant angle of a degrees.
- **shuffle(ps)** shuffles the widgets of the pictures of ps. More precisely, given pictures p1=[w11,...,w1n1],...,pk=[wk1,...,wknk], `shuffle` draws the widgets in the order w11,...,wk1,...,w1n1,...,wknk.
- **slice(r,a)** and **slices(r,a,c)** displays the outline resp. c-colored plane of a slice with opening angle a of a circle with radius r.
- **snow(n,r)** displays the outline of a Koch snowflake with depth n and radius r.
- **snow(n,r,c)** displays a c-colored Koch snowflake with depth n and radius r. The triangles that build up the snowflake are colored differently, depending on the level where they are created. For each two successive levels l and l', the color of l is the successor of the color of l' with respect to a circular list of n equidistant colors.
- **spline[ps]** and **spline([ps],c)** display B-splined versions of `path[ps]` and `path([ps],c)`, respectively. `pathS[ps]` and `pathS([ps],c)`, respectively, yield the same results, but call Tcl/Tk's built-in splining algorithm.
- **splineC[ps]**, **splineC([ps],c)** and **splineF([ps],c)** display B-splined versions of `poly[ps]`, `poly([ps],c)` and `polyF([ps],c)`, respectively. `polyS[ps]`, `polyS([ps],c)` and `polySF([ps],c)`, respectively, yield the same results, but call Tcl/Tk's built-in splining algorithm.
- **split(ps)** extracts the widgets enclosed in a turtle widget of ps and draws them on top of each other.
- **splitS(sc,ps)** inserts the widgets enclosed in turtle widgets of ps into ps, draws them at their original places and *scales* the resulting picture with factor sc. The spacing between the extracted widgets is preserved only if sc coincides with the actual scale factor (see the *paint buttons*).
- **squareA/B(d,ps)** displays `tabA/B(n,d,ps)` (see below) where n is the square root of the number of widgets of ps.
- **star(n,r,r')** and **star(n,r,r',c)** displays an the outline resp. c-colored plane of a star with n peaks such that the maximum of r and r' is the peak radius and the minimum is the valley radius of the star.
- **tabA(n,d,ps)** combines the widgets of ps to a single one and displays them as a matrix with n columns and a space of d pixels between the *anchors* of adjacent widgets.
- **tabB(n,d,ps)** combines the widgets of ps to a single one and displays them as a matrix with n columns and a space of d pixels between the *borders* of adjacent widgets. (The vertical space is determined by the greatest widgets of adjacent rows.)
- **taichi(yin,yang,c)** displays a tai chi symbol with the (left) yin part colored in c and the (right) yang part colored in the complement d of c. Moreover, the text `yin` is entered into the yin part in color d and the text `yang` is entered into the yang part in color c.
- **text(s)** and **text(s,c)** display the string s in black resp. color c. The text font and size must be set in the window of the solver from which the painter was called.
- **tree(t)** and **tree(t,c)** display the tree t with black resp. c-colored node entries and blue edges. Pointers are not dereferenced!
- **tria(r)** and **tria(r,c)** display the outline resp. c-colored plane of an equilateral triangle with peak radius r.
- **trunk** displays `trunk(blue)`.
- **trunk(c)** displays the c-colored trunk of a Pythagorean tree.
- **turt(acts)** displays the picture a turtle draws when starting in `state0` (position (0,0), orientation 0, color black, lightness value 0) and executing the actions of acts sequentially. These are the possble turtle actions:
    - **M(d)**: Move a distance of |d| pixels. If d is positive, move forward. If d is negative, move backward. The turtle draws a line from its old to its new position. At the end of an action sequence, the turtle combines these lines to paths or polygons. The shape and color depends on entries in the stack of states each of which is a sixtuple (p,a,c,n,sc,ps,w) consisting of a position p, an orientation a, a color c, a shape value n for the path still to be drawn, a scaling factor sc, a list ps of points to be connected and the widget w painted at last.
    - **J(d)**: Jumps |d| pixels. Same as `M(d)`, but no line is drawn.
    - **T(a)**: Turn by a degrees.
    - **L**: Turn left. Returns the value of `T(-90)`.
    - **R**: Turn right. Returns the value of `T(90)`.
    - **B**: Turn backwards. Returns the value of `T(180)`.
    - **O**: Open a new subpicture. Equals `O(black)`.
    - **O(c)**: Open a c-colored path. Given the stack top (p,a,c',n,sc,ps,w), the turtle pushes (p,a,c,0,sc,[p],Nothing) on top of the stack. The path drawn upon closing will be c-colored.
    - os displays `OS(black)`.
    - **OS(c)**: Open a c-colored smooth path. Given the stack top (p,a,c',n,sc,ps,w), the turtle pushes (p,a,c,1,sc,[p],Nothing) on top of the stack.
    - **OF** displays `OF(black)`.
    - **OF(c)**: Open a c-colored polygon. Given the stack top (p,a,c',n,sc,ps,w), the turtle pushes (p,a,c,2,sc,[p],Nothing) on top of the stack.
    - **OFS** displays `OFS(black)`.
    - **OFS(c)**: Open a c-colored smooth polygon. Given the stack top (p,a,c',n,sc,ps,w), the turtle pushes (p,a,c,3,sc,[p],Nothing) on top of the stack.
    - **SC(sc)**: Open a scaled subpicture. Given the stack top (p,a,c,n,sc',ps), the turtle pushes (p,a,c,n,sc*sc',ps) on top of the stack.
    - **C**: Close a subpicture. Given the stack top (p,a,c,n,sc,ps,w), The turtle pops the stack and returns to the state that is now on top of the stack. Moreover, it draws a path connecting the points it has visited since the last opening action. The shape and the color of the path are determined by that action (see above).
    - **D**: Draw. The turtle draws the path just described, but does not pop the stack. It only removes the connected points from the stack top. Hence, in contrast to `C`, the turtle does not return to its position, orientation, etc. that it took during the last opening action.
    - Any widget w recognized by *polygon*: Given the stack top (p,a,c,n,sc,ps,v), the turtle draws w at position p with orientation a and scaling factor sc and replaces v by w.
- **turt(ps)** combines the widgets of ps to a single one and displays them on top of each other.
- **wave(n,d,a,c)** displays a c-colored wave with n peaks, a slope length of d pixels and a gradient angle of a degrees.
- **waveS(n,d,a,c)** displays a c-colored smooth wave with n peaks, a slope length of d pixels and a gradient angle of a degrees.
- If c is generated by the color grammar (see the *Grammar*), then **c(ps)** colors the widgets of ps with the color denoted by c.

polygon solution recognizes each solved formula

```
 Any Z1:x1=t1 &...& Any Zk:xk=tk & All Z(k+1):x(k+1)=/=t(k+1) &...& All Zn:xn=/=tn
```

and submits the terms t1,...,tn to *polygon*.

polygons combines the maximal subgraphs of the current graphs that are recognizable by *polygon*.

rectangles interprets a term of the form `[(x1,y1,b1,h1),...,(xn,yn,bn,hn)]` as a collection of rectangles r1,...,rn such that for all 1 <=

i <= n, (xi,yi) is the top-left corner, bi the breadth and hi is the height of ri.

If a file F contains the Haskell code of a graph G, each interpreter compiles the term **`file(F)`** into G. Conversely, given a term t that represents a graph G, if the command *store graph* is applied to the term `file(t,F)`, it saves the Haskell code of G to F and replaces `file(t,F)` by `file(F)` (see the graph menu).

Alignments and palindromes

This and the following sections deal with the alignment, palindrome, dissection and partition enumerators that can be called from the solver's term/formula menu. Other enumeration algorithms may be added accordingly. The alignment enumerator and the palindrome enumerator compute alignments between two string sequences [Gie] or within a single sequence [GM], respectively. A development of the Haskell program for the former can be found in [P01], Section 2.4.

After two lines xs and ys of strings separated by blanks have been entered into the text field, the **alignment enumerator** asks for a constraint. There are two possibilities:

- **match**. The alignment enumerator computes all syntax trees for xs#reverse(ys) with a maximal number of `equal`- or `compl`-nodes according to the following grammar G:

  ```
   match : align ---> match

  insert : align ---> insert

  delete : align ---> delete

  end : align ---> #

  equal : match ---> s align s   for all strings s

  compl : match ---> s align compl(s)   for all strings s

  ins : insert ---> align s   for all strings s

  del : delete ---> s align for all strings s
  ```

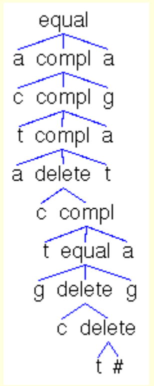
  compl is a function on strings defined by a specification that must be entered before the enumerator is called. The specification is also supposed to contain an equational axiom `labels=t` where t simplifies to a list of constants. *compl* maps all strings that are not in *labels* to # (see, e.g., ALIGN).
- **local**. The alignment enumerator computes all syntax trees for xs#reverse(ys) with a *maximal local alignment*, i.e. a path consisting of `equal`- or `compl`-nodes, according to G.

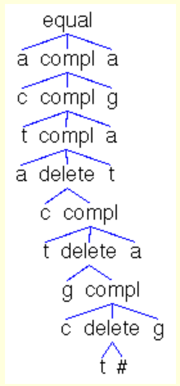
Following the assignment of complementary DNA bases, the function *compl* maps a to t, t to a, c to g, g to c and all other strings to #. Corresponding axioms are loaded when the alignment or palindrome enumerator is called from the solver.

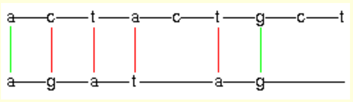
Given the sequences

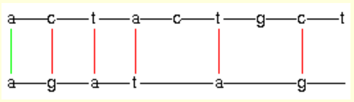
```
 s1 = a c t a c t g c t, s2 = a g a t a g,

s3 = a d f a a a a a a, s4 = a a a a a a d f a,
```

the trees in Fig. 25 are the only two syntax trees of s1#reverse(s2) that meet the *match*-constraint and the only two syntax trees of s3#reverse(s4) that meet the *match*-constraint and the *local*-constraint, respectively. Here *compl* is defined by ALIGN.









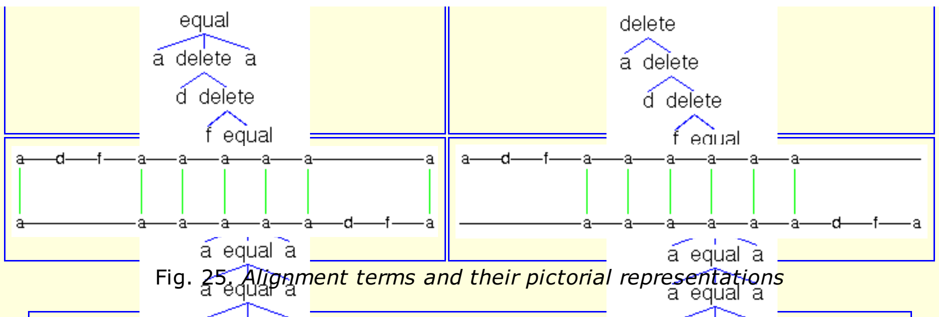
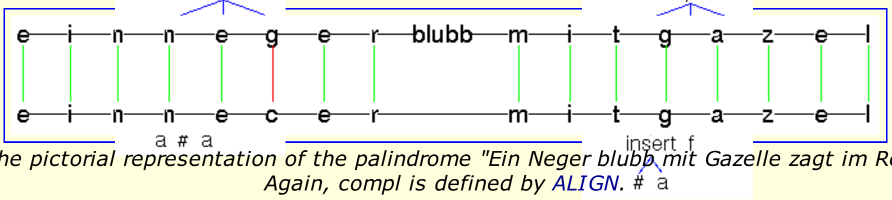


Fig. 25. *Alignment terms and their pictorial representations*


e i n n e g e r blubb m i t g a z e l
e i n n e c e r m i t g a z e l
a # a
insert f
# a
Fig. 26. *The pictorial representation of the palindrome "Ein Neger blubb mit Gazelle zagt im Recen nie". Again, compl is defined by* ALIGN*.*

After a sequence xs of strings separated by blanks has been entered into the text field, the **palindrome enumerator** computes syntax trees for xs with a maximal number of `equal`- or `compl`-nodes according to grammar G above with *end* rule replaced by two rules:

```
 single : match ---> s for all strings s

end : align ---> _
```

Moreover, the *match* rule is preferred to the *insert* and *delete* rules. Again, *compl* is a function on strings defined by a specification that must be entered before the enumerator is called (see above).

Dissections and partitions

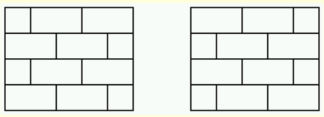


Fig. 27. *All dissections of a 5x4-rectangle that satisfy area(1,2)&brick&hori: each dissection satisfies brick and consists of subrectangles covering 1 or 2 unit squares and satisfying hori (see below).*

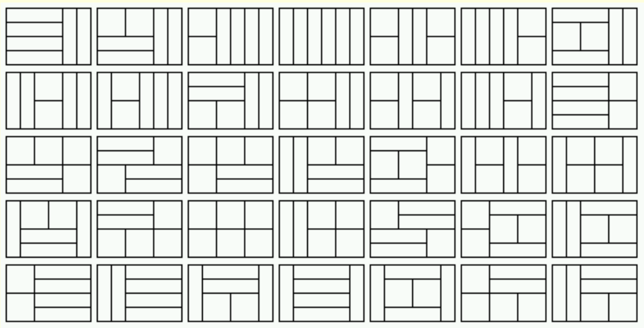


Fig. 28. *All dissections of a 6x4-rectangle that satisfy eqarea(6): each dissection consists of 6 subrectangles that cover the same area.*

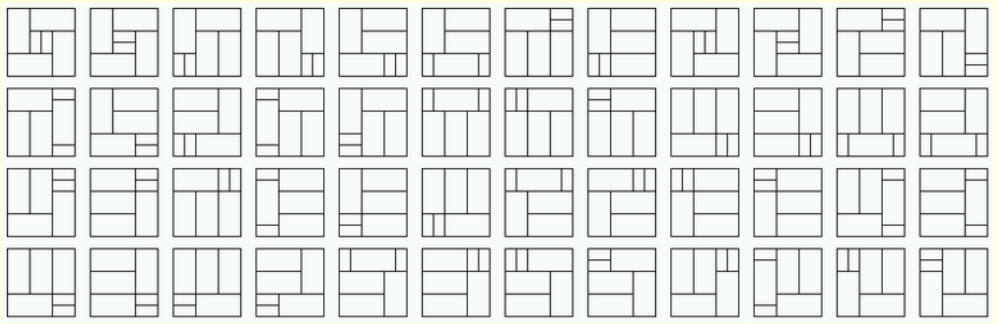


Fig. 29. *All dissections of a 6x6-rectangle that satisfy sizes[6]&factor(2): each dissection consists of 6 subrectangles and the breadth b and the height h of each subrectangle satisfy b=2*h or h=2*b.*

The **dissection enumerator** computes dissections of a rectangle and represents them directly without a detour via term representations. The underlying algorithm creates and modifies a triple of lists of top, left and inner subrectangles, respectively, such that dissection elements violating certain given constraints are discarded as early as possible (see [P94], Section 4).

Constraints. The dissection enumerator returns dissections of a given rectangle with breadth b and height h that satisfy one of the following atomic constraints or disjunctive or conjunctive combinations thereof:

constraint	holds true for all dissections
area(n)	consisting of ceiling((b*h)/n) subrectangles that cover at most n unit squares

area(m,n)	consisting of subrectangles that cover at least m and at most n unit squares
brick	consisting of subrectangles r such that for all (x,y,b,h),(x,y',b',h') ∈ r, x=0, y'=/=y+h y' or y=/=y'+h' (see below)
eqarea(n)	consisting of n subrectangles that cover the same number of unit squares
factor(p)	consisting of subrectangles such that the breadth b and the height h of each subrectangle satisfy b=p*h or h=p*b
hori	consisting of subrectangles whose height does not exceed the breadth
sizes(ns)	consisting of n ∈ ns subrectangles
True	
vert	consisting of subrectangles whose breadth does not exceed the height

Formulas built up of atomic constraints are parsed according to the *Grammar*. Each constraint is translated into a triple of the Haskell type

```
 ((Int,Int,Int,Int) -> Bool, [Int], [(Int,Int,Int,Int)] -> [(Int,Int,Int,Int)] -> Bool).
```

The first component is a Boolean function that checks individual rectangles each of which is represented as a quadruple (x,y,b,h) where (x,y) is the top-left corner, b the breadth and h the height of the rectangle.
The second component lists the admissable cardinalities of a dissection. The third component is a Boolean function that checks a relation between two parts of a dissection. Such a Boolean function is needed for expressing the brick constraint.

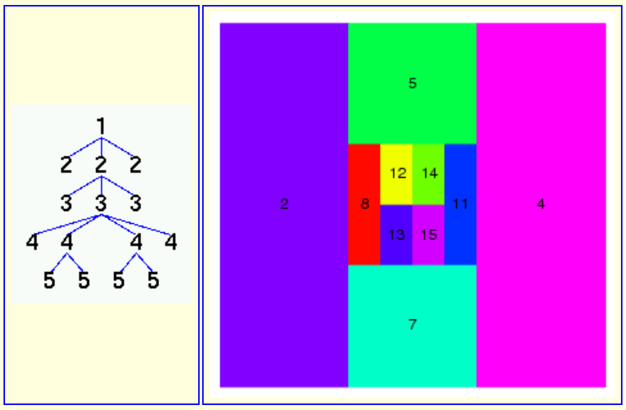


Fig. 30. *A nested partition satisfying eqout&sym of a list with 10 elements and its interpretation by partition (see the pict type menu).*

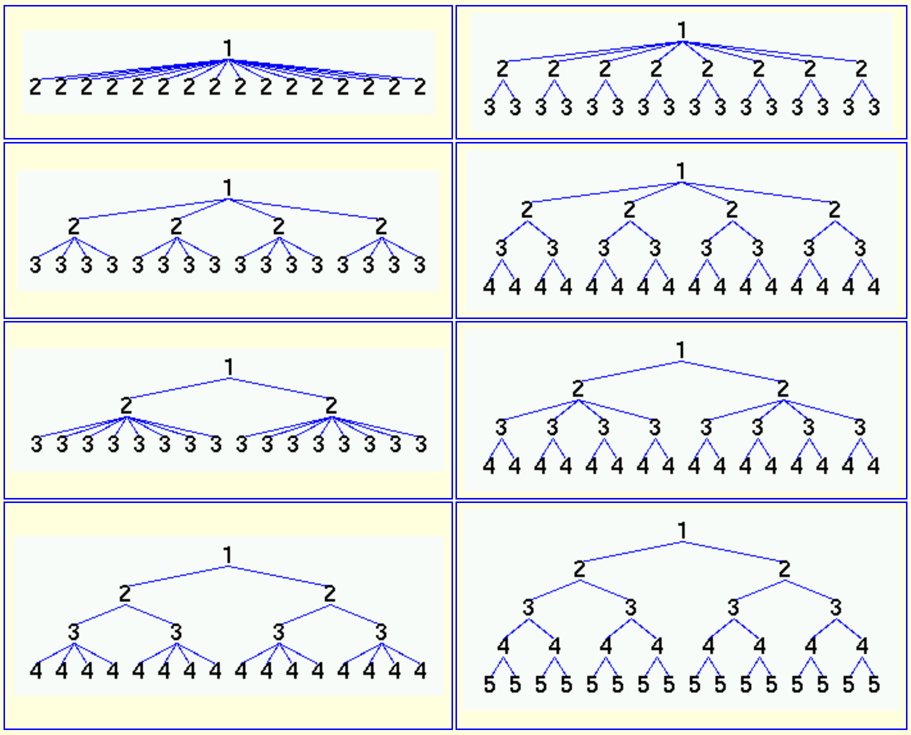


Fig. 31. *The nested partitions satisfying bal&eqout of a list with 16 elements*

The **partition enumerator** computes nested partitions of a list and represents them as trees whose nodes are labelled with the nesting degrees of the respective subpartitions. Partitions with singleton subpartitions are not constructed.

Let s be a set with n elements and parts(s) be the set of non-nested partitions of s with at least two elements. A Haskell program that computes the cardinality f(n) of parts(s) reads as follows:

```
 f 0 = 1

f n = sum (map g [0..n-1]) where g i = (fact n/(fact (n-i)*fact i))*f i

fact i = product [1..i]
```

For the number h(n) of *nested* partitions of s we obtain:

```
 h 2 = 1

h n | n > 2 = sum[product[h (length p) | p <- ps] | ps <- parts s]
```

Hence, without meeting additional constraints, the number of trees representing nested partitions increases combinatorially with the number of leaves:

number of leaves	**number of trees**
5	45
6	197
7	903
8	4279
9	20793
10	103049

Constraints. The partition enumerator returns nested partitions that satisfy one of the following atomic constraints or disjunctive or conjunctive combinations thereof.

constraint	**holds true for all trees**
alter	whose nodes at even (odd) positions of a list s of all nodes with the same direct predecessor are leaves (inner nodes) unless s consists of leaves
bal	that are balanced
eqout	whose inner nodes with the same direct predecessor have the same outdegree
hei(n)	whose height is at most n
levmin	whose inner nodes at level n have an outdegree of at least n
levmax	whose nodes at level n have an outdegree of at most max(2,n)
sym	that are vertically symmetric
out(m,n)	whose inner nodes at level n > 1 have an outdegree between m and n
True	

Formulas built up of atomic constraints are parsed according to the *Grammar*.

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